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The sub-dimensions of metacognition and their influence on modeling competency

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Mathematical modeling is indeed a versatile skill that goes beyond solving real-world problems. Numerous studies show that many students struggle with the intricacies of mathematical modeling and find it a challenging and complex task. One important factor related to mathematical modeling is metacognition which can significantly impact expert and student success in a modeling task. However, a notable gap of research has been identified specifically in relation to the influence of metacognition in mathematical modeling. The study's main goal was to assess whether the different sub-dimensions of metacognition can predict the sub-constructs of a student's modeling competence: horizontal and vertical mathematization. The study used a correlational research design and involved 538 participants who were university students studying mathematics education in Riau Province, Indonesia. We employed structural equation modeling (SEM) using AMOS version 18.0 to evaluate the proposed model. The measurement model used to assess metacognition and modeling ability showed a satisfactory fit to the data. The study found that the direct influence of awareness on horizontal mathematization was insignificant. However, the use of cognitive strategies, planning, and self-checking had a significant positive effect on horizontal mathematization. Concerning vertical mathematization, the direct effect of cognitive strategy, planning, and awareness was insignificant, but self-checking was positively related to this type of mathematization. The results suggest that metacognition, i.e., awareness and control over a person's thinking processes, plays an important role in modeling proficiency. The research implies valuable insights into metacognitive processes in mathematical modeling, which could inform teaching approaches and strategies for improving mathematical modeling. Further studies can build on these findings to deepen our understanding of how cognitive strategies, planning, self-assessment, and awareness influence mathematical modeling in both horizontal and vertical contexts.

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Introduction

Changing curriculum content and instructional styles in teaching and learning processes for regular mathematics classes is critical to promote more meaningful engagement with mathematics (Schoenfeld, 2016). A shift to searching for solutions, exploring patterns, and formulating conjectures rather than simply memorizing procedures and formulas or completing exercises can lead to deeper understanding and more versatile problem-solving skills. Incorporating mathematical modeling into classroom activities by engaging students in authentic problem-solving within complex systems and interdisciplinary contexts can help develop the competencies to tackle increasingly complex problems. Mathematical modeling can strengthen problem-solving skills and connect mathematics to real-world situations, making it relevant to students' current and future lives (Hidayat and Wardat, 2023). The importance of mathematical modeling is further underscored by its inclusion as a primary component in the mathematics assessment of the Program for International Student Assessment (PISA) (Niss, 2015). Students can tackle non-routine real-life challenges by engaging in modeling activities and working collaboratively on realistic and authentic mathematical tasks. However, traditional instructional methods for assessing student modeling proficiency are inadequate. This information underscores the need for improved methods of evaluation that capture the full range of students' modeling abilities and the development of their problem-solving skills. Educators should consider incorporating alternative assessment methods such as project-based assessments, performance tasks, or reflective journals to better assess student modeling skills. In addition, professional development opportunities for teachers to learn effective strategies for integrating mathematical modeling into their instruction can contribute to more successful implementation and assessment of these skills.

Mathematical modeling is a multifaceted skill beyond solving real-world problems (Mohd Saad et al., 2023; Niss et al., 2007). As Minarni and Napitupulu (2020) point out, students can apply modeling abilities to describe context problems mathematically, organize tools, discover relationships, transfer between real-world and mathematical problems, and visualize problems in various ways. In modeling real-world problems, students activate other competencies, such as representing mathematical objects, arguing, and justifying (National Council of Teachers of Mathematics, 1989). Engaging in mathematical modeling in the classroom helps students clarify and interpret phenomena, solve problems, and develop social competencies necessary for effective teamwork and collaborative knowledge building. Mathematical modeling instruction aims to improve students' mathematical knowledge, promote critical and creative thinking, and foster positive attitudes toward mathematics (Blum, 2002). Cognitive modeling combined with task orientation is more effective in increasing the likelihood of success. In high school curricula, students can connect mathematical modeling to different courses, reinforcing the importance of this skill in different contexts (Hernández et al., 2016). Integrating mathematical modeling into different subject areas can help students develop a comprehensive understanding of the relevance and applicability of mathematics in real-world situations, ultimately leading to better problem-solving abilities and an appreciation for the power of mathematical thinking.

Numerous studies have shown that mathematical modeling is challenging for many students (Anhalt et al., 2018; Corum and Garofalo, 2019; Czocher, 2017; Kannadass et al., 2023). Metacognitive competencies improve students' modeling abilities (Galbraith, 2017; Vorhölter, 2019; Wendt et al., 2020). Metacognition, the ability to reflect on and regulate one's thinking, can significantly impact expert and student success in problem-solving (Schoenfeld, 1983, 2007). Productive metacognitive behaviors can help students better understand the given problem, search for and distinguish relevant and irrelevant information, and focus on the overall structure of the problem (Kramarski et al., 2002). These behaviors can lead to improved understanding and problem-solving abilities. Although the benefits of metacognition to learning are widely recognized, there is limited research on the specific types of metacognitive strategies that are most effective in helping students (Wilson and Clarke, 2004). Future research should focus on identifying these strategies and understanding how they can best be used in educational settings to improve students' mathematical modeling and problem-solving abilities. This research could include exploring the most effective methods for teaching metacognitive skills, examining how metacognition can be tailored to individual student needs, and examining the impact of metacognitive interventions on student modeling performance. Thus, this study aimed to investigate how the sub-dimensions of metacognition can predict modeling performance. The study questions are as follows: (a) Do the sub-constructs of metacognition (awareness, cognitive strategy, planning, and self-checking) predict horizontal mathematization? (b) Do the sub-constructs of metacognition (awareness, cognitive strategy, planning, and self-checking) predict vertical mathematization?

Theoretical perspective

Models and modeling perspective (MMP). The term 'model' is a collection of elements, connections between elements, and actions that describe or explain how the elements interact (English, 2007; Lesh and Doerr, 2003). Modeling exercises allow students to reveal their multiple forms of reasoning, create conceptual frameworks, and develop effective ways to represent the structural features of the topic (Carreira and Baioa, 2018). Models and Modeling Perspective (MMP), also known as contextual modeling (Kaiser and Sriraman, 2006), is considered a method to understand real-life situations and develop formal mathematical knowledge based on students' understanding (Csapó and Funke, 2017; Lesh and Doerr, 2003). Students must move from a real-world situation to a mathematical world using their previously learned mathematical concepts as a modeling tool that goes beyond calculational prescriptions (Sevinc, 2022) and learning theories (Abassian et al., 2019). Moreover, MMP considers the mathematical model as a conceptual tool of a mathematical system that emerges from a specific real-world situation (Lesh and Lehrer, 2003). In brief, MMP is a new concept that incorporates real-world context into the teaching and learning of mathematical problem-solving because MMP prepares students to be mentally active in modeling. An important feature of MMP is the recognition that problem-solving typically involves numerous modeling cycles in which descriptions, explanations, and predictions are continuously refined. In contrast, solutions are modified or discarded depending on their interpretation of the world.

Students will use their internal conceptual systems to organize, understand, and make connections between events, experiences, or issues (Erbas et al., 2014) to adapt to MMP. Student learning through the use of MMP will also facilitate communication between peers and teachers through project-based learning or problem-based learning (Ärlebäck, 2017) as they practice solving authentic problem situations by engaging in mathematical thinking that involves interpreting situations, describing and explaining, computing through procedures, and deductive reasoning (English et al., 2008). MMP summarizes a cycle of activities that, in the first step, requires students to understand the real-world situation, followed by structuring the situation model, mathematizing to develop a mathematical model, and collaborating mathematical models to develop results that are considered and validated within the real-world situation, and finally presenting a solution to a real-world situation.

Mathematical modeling and mathematization. Modeling is also known as organizing representative descriptions in which symbolic representations and formal model structures develop (Hidayat et al., 2018; Niss, 2015). According to the South African Department of Basic Education (2011), mathematical modeling is an important curriculum focus, and real-world situations should be included in all areas, such as economics, health, social services, and others. Mathematical modeling is a process of mathematization or mathematization in which students can discover relevant issues or assumptions in a given real-world scenario by mathematizing, interpreting, and evaluating solutions to resulting mathematical problems related to the given circumstance (Leong and Tan, 2020). The mathematization method can be applied as a series of activities directed toward the activity system object, with the goal of the modeling project serving as the activity object itself (Araújo and Lima, 2020). Students with mathematical skills can acquire mathematical knowledge through logical reasoning using problem-solving. Formal mathematical information is obtained during the mathematization process by referring to informal knowledge, including components of actual problem situations (Freudenthal, 2002). Mathematical modeling can be divided into many tasks: simplifying, mathematizing, computing, interpreting, and validating. When students are proficient in the modeling process, they can independently and insightfully perform all components of a mathematical modeling process (Hankeln et al., 2019), with the focus of the competencies being on identifying specific fundamental capabilities.

A mathematical model is created using mathematization (Yilmaz and Dede, 2016). The concept of mathematization involves using mathematical methods to organize and examine various aspects of reality. The idea of the mathematization of actual reality is formulated in two forms of mathematization (Treffers, 1978; Treffers and Goffree, 1985), namely horizontal and vertical mathematization. Horizontal and vertical mathematization are complementary processes in mathematical modeling and problem-solving (Freudenthal, 1991). The process of horizontal mathematization begins with understanding the problem and extends to problem-solving (Galbraith, 2017). Horizontal mathematization involves translating real-world problems into mathematical representations, while vertical mathematics involves working within mathematics to solve the problem. Both processes are important for students to develop a comprehensive understanding of mathematics and its applications in real-world situations. Horizontal mathematization refers to translating a real-world problem into a mathematical problem or representation. Students identify relevant mathematical structures, concepts, and relationships related to the given problem in this phase. They may simplify the problem by making assumptions, recognizing patterns, or constructing a model. Horizontal mathematization aims to create a mathematical representation that captures the essence of the real-world situation and can be analyzed using mathematical tools. Simplification is about understanding the core problem and using mathematics to construct a model based on reality (Kaiser and Schwarz, 2006). Students must be able to clarify the essential elements of the situation, formulate the problem, and create a simplified version that can be analyzed mathematically. A further step is to identify relevant mathematical concepts, variables, and relationships that capture the essence of the real situation (mathematization). Students must be able to translate the problem into mathematical language using appropriate notations or visual representations (Kaiser and Stender, 2013). This study defines horizontal mathematization as simplifying assumptions, clarifying the objective, formulating the problem, assigning variables, establishing parameters and constants, formulating mathematical expressions, and selecting a model (Yilmaz and Dede, 2016).

Vertical mathematization occurs after the problem has been translated into a mathematical representation through horizontal mathematization. In this phase, students work within the domain of mathematics to solve the problem by using mathematical techniques, calculations, proofs, or manipulations. Vertical mathematization is about delving deeper into mathematical concepts, exploring connections, and gaining new insights. The focus here is on applying mathematical knowledge and reasoning to find a solution to the problem. Vertical mathematization refers to exploring the realm of formal symbols (Selter and Walter, 2019). Vertical mathematization also refers to the mathematical processing and improvement of real-world problems transformed into mathematics (Treffers and Goffree, 1985). Learners apply their mathematical knowledge or intuitive procedures to solve the problem within the framework of the mathematical model (Maaß, 2006). This model may involve calculations, manipulations, or proof to derive a mathematical solution. Once a mathematical solution is found, students must interpret the results in the context of the original problem (Garfunkel and Montgomery, 2016). To do this interpretation, they must understand the relationship between the mathematical solution and the real-world situation and place the solution in terms of the problem's context. The final step is to review the solution for accuracy and critically evaluate the assumptions made, the model used, and the overall process (Kaiser and Stender, 2013). Students must determine if their solution is reasonable and sensible and if improvements or changes can be made to the model or assumptions. This paper defines vertical mathematization as interpreting, validating, and relating the result to a real-world context.

Metacognition. Metacognition encompasses two aspects: the capacity to recognize and understand one's cognitive processes (referred to as metacognitive knowledge) and the ability to manage and adapt these processes (known as metacognitive control) (Fleur et al., 2021). This study must consider metacognition because modeling issues are typically worked on in small groups (Biccard and Wessels, 2011). Metacognition includes students' understanding of their cognitive processes and their capability to regulate and manipulate them (Kwarikunda et al., 2022). Metacognition is the knowledge or cognitive activity that targets or controls any component of a cognitive effort (Flavell, 1979); for example, students use metacognition to solve issues while studying. Students must manage their cognitive processes during learning so that their learning achievement be measured afterward (Bedel, 2012). Metacognition is often divided into two parts: metacognitive knowledge and techniques, which are often complemented by an affective-motivational aspect (Efklides, 2008; Veenman et al., 2006). Planning cognitive activities, monitoring progress toward goals, selecting methods to solve difficulties, and reflecting on past performance to improve future outcomes are all examples of metacognitive techniques (Kim and Lim, 2019). Furthermore, O'Neil and Abedi (1996) operationalize students' metacognitive inventory as a construct that includes planning, self-checking, cognitive strategy, and awareness. Metacognition is understanding how individuals gain information and manage the process (Schraw and Dennison, 1994).

Metacognitive abilities have a significant impact on student learning and performance. They enable students to identify areas of difficulty and select appropriate learning strategies to understand new concepts. Metacognition has been found to improve students' problem-solving abilities (García et al., 2016). However, metacognitive skills differ among students with varying levels of modeling competence, with some putting little effort into organizing or expressing knowledge differences (García et al., 2016). Students with high levels of modeling competence tend to pay more attention to time management, which may contribute to their success in problem-solving tasks. Interestingly, metacognitive training is particularly beneficial for lower-performing students because it allows them to improve while working on the same tasks as their peers (Karaali, 2015). This finding suggests that metacognitive instruction can help level the playing field for students with different abilities and allow all learners to develop their problem-solving skills more effectively. In summary, metacognition is critical in mathematics and affects students' abilities differently. Educators should integrate metacognitive training into their instructional practices to support all learners and help them develop self-awareness, reflection, and regulation skills to benefit their mathematical problem-solving efforts.

Relationship between metacognition and modeling competency. Metacognition can help with goal-oriented modeling and overcoming various challenges (Stillman, 2004), depending on students' knowledge and experience. The success of metacognitive activity can be attributed to students' responses to specific problem-solving scenarios that can activate metacognition (Vorhölter, 2021). Metacognition is an essential method associated with mathematical proficiency and problem-solving skills. Teachers can help students develop appropriate individual techniques for dealing with modeling challenges and various metacognitive activities, such as mathematizing across different circumstances and environments (Blum, 2011). Mathematizing is a horizontally sequential process of translating parts of the real world into the language of symbols and abstracting in a vertical direction (Freudenthal, 2002). The mathematization process is horizontal mathematization because it requires the learner to transform real life into mathematical symbols. Horizontal mathematization leads to results based on different problem-solving strategies and the concrete problem case (Gravemeijer, 2008). The process of horizontal mathematization focuses primarily on organizing, schematizing, and constructing a model of reality so that it can be treated mathematically (Piñero Charlo, 2020). Horizontal mathematization is highlighted as a learning difficulty in an instructional strategy where teachers do not recognize horizontal mathematization as a learning problem (Yvain-Prébiski and Chesnais, 2019), and students also have difficulty discovering connections and transferring real-world problems to known mathematical models. Changing models, merging and defining a connection in a formula, and improving and integrating models are challenges of vertical mathematization (Suaebah et al., 2020). Real-world modeling activities that promote the horizontal mathematization process can help students experience mathematics as a value by strengthening their understanding and tangible connection between mathematics and the effort expended, i.e., by improving their metacognition skills (Suh et al., 2017).

Awareness of metacognition is critical in developing and improving students' problem-solving skills. Studies have shown a significant positive correlation between metacognition awareness and problem-solving abilities (Sevgi and Karakaya, 2020). Effective mathematical problem-solving is also associated with planning and revision techniques (García et al., 2019). Students can improve their problem-solving skills through self-reflection on planning, monitoring, and evaluating their thinking processes (Herawaty et al., 2018). This finding highlights the link between metacognition and modeling abilities such as awareness, self-checking, planning, and cognitive strategy. By using planning techniques, students can improve their problem-solving abilities, for example, through verbalization (Zhang et al., 2019). Although the transfer of metacognitive knowledge to mathematical modeling is modest, using planning and revision procedures still contributes positively to student success. The sub-dimension of monitoring can predict a student's engagement in a discussion (Akman and Alagöz, 2018). Using cognitive strategies during the formulation phase of the modeling process provides a sense of guidance (Krüger et al., 2020). Awareness of metacognition and using metacognitive strategies such as planning, monitoring, and revising are essential to improve students' problem-solving and mathematical modeling abilities. Educators should aim to incorporate metacognitive strategies into their teaching methods to support the development of these skills in students.

Metacognition has been recognized as critical for solving complicated tasks, such as modeling tasks (Wilson and Clarke, 2004). Individuals can cultivate a more methodical and comprehensive approach to horizontal mathematization by integrating the sub-constructs of metacognition (awareness, planning, self-checking, and cognitive strategies). For example, horizontal mathematization is enhanced by providing students with useful tools and tactics for planning, analyzing, and solving modeling tasks through awareness, planning, self-checking, and cognitive strategies. Students can recognize mathematical patterns and structures within a modeling task when they know the relevance and use of mathematics in everyday situations. Creating a plan allows students to break difficult tasks into manageable parts. Students can be disciplined and avoid errors or omissions by setting goals, outlining necessary mathematical operations, and choosing a sequence of tasks. Cognitive techniques enable effective information processing, allow students to connect different mathematical ideas, and promote creative thinking when solving modeling tasks. Finally, self-checking promotes error detection and correction, leading to a better understanding of mathematical ideas. At the same time, the sub-constructs of metacognition (awareness, planning, self-checking, and cognitive strategies) would help enhance vertical mathematization skills. For example, students can identify the relevant mathematical relationships and structures needed to build a mathematical model by improving their awareness. To fulfill this aim, they must recognize the mathematical concepts and principles that apply to the current real-world problem. Again, the objectives are set in the planning phase, variables and parameters are selected, and the mathematical operations and transformations are described. The problem is analyzed using cognitive techniques, and the mathematical solution is found through reasoning, pattern recognition, and visualization. Finally, self-validation assures that the mathematical model is accurate and reliable. Students can locate any errors or inconsistencies and correct them by examining and checking the model frequently.

Hypotheses. The hypotheses of the research are as follows:

i. Significant relationships will occur between awareness and horizontal mathematization.
ii. Significant relationships will occur between cognitive strategy and horizontal mathematization.
iii. Significant relationships will occur between planning and horizontal mathematization.
iv. Significant relationships will occur between self-checking and horizontal mathematization.
v. Significant relationships will occur between awareness and vertical mathematization.

In formulating a simple mathematical model for determining the optimal placement of a bus stop on a new bus route, which of the following assumptions is considered the least significant?
(a) Assuming that only one bus shelter will be constructed;
(b) Assuming that the road is linear and without curves;
(c) Assuming that the probability of dry weather is twice as likely as that of wet weather;
(d) Assuming that the bus operates on a schedule of one bus every thirty minutes and;
(e) Assuming that customers are unwilling to walk long distances to access a bus.

Fig. 1 The examples of horizontal mathematization test.

vi. Significant relationships will occur between cognitive strategy and vertical mathematization.
vii. Significant relationships will occur between planning and vertical mathematization.
viii. Significant relationships will occur between self-checking and vertical mathematization.

Methodology

Participants and design. This study used a correlational research design (Creswell, 2012; Shanmugam and Hidayat, 2022), which explores the level of interrelation between metacognition and mathematical modeling using structural equation modeling (SEM). The current study sample consisted of college students studying mathematics education in Riau Province, Indonesia, with similar modeling experiences. These students were prospective mathematics teachers who were prepared to teach mathematics at the secondary level. First-year (133 or 24.7%), second-year (223 or 41.4%), and third-year (182 or 33.8%) students participated in the study, with a total of 538 samples. The fourth-year study samples were not included due to practical exercises. All participants were selected using cluster random sampling from universities with similar characteristics such as location and modeling experience. We used this type of sampling because this research focused on groups rather than individuals, which resulted in students coming from selected universities to take the test. Although the current research found that the percentage of gender resulted in more female (483 or 89.8%) than male (55 or 10.2%) samples, we did not use gender as a moderator or covariate for analyzing the data. The Department of Investment and Integrated One Stop Services, Indonesia, approved the study. Subsequently, all selected samples received written informed consent. We explained the study's objectives and the voluntary nature of participation before the test was administered. All students from the selected universities took 60 min to complete the metacognitive inventory instrument and the mathematical modeling test.

Measures. To measure mathematical modeling competence, we developed and used the Modeling Test (Haines and Crouch, 2001), which we divided into two sub-constructs: horizontal and vertical mathematization. The items were assessed by multiple-choice questions with a three-level scoring (0=wrong answer, 1=partially correct answer, and 2=true answer). The modeling test had 22 questions and a final score of 44. Moreover, the test is also suitable for this study because the study included a large sample (Lingefjärd and Holmquist, 2005). Figure 1 shows one of the examples of measuring horizontal mathematization.

Reliability scores for modeling competence followed the sub-construct: horizontal mathematization (18 items, $\alpha = 0.861$) and vertical mathematization (4 items, $\alpha = 0.740$). These overall reliability values were acceptable ($\alpha > 0.70$) (Tavakol and Dennick, 2011). The internal consistency of the mathematical modeling test was good, with composite reliability values (CR) ranging from 0.775 to 0.925 (> 0.6). The value of the Average Variance Extracted (AVE) ranged from 0.500 to 0.501 (> 0.5), indicating good discriminant validity. At the same time, the square roots of all AVE values were larger than the associations suggested among them or to the left of them, which underlined the discriminant validity of the mathematical modeling test. All these values were consistent with the recommendations of researchers (Fornell and Larcker, 1981; Hair et al., 2010; Nunnally and Bernstein, 1994), which were satisfactory.

The metacognitive inventory (O'Neil and Abed, 1996) was adopted for measuring metacognition, which comprised four sub-scales: awareness (5 items), cognitive strategy (5 items), planning (5 items), and self-checking (5 items). The example of the item for each sub-contract provided (awareness; *I am always aware of my thoughts in modeling task*), (cognitive strategy; *I am trying to find the main idea in the modeling task*), (planning; *I am trying to understand the purpose of the modeling task before attempting to solve it*) and (self-checking; *If I notice any mistakes while working on the modeling task, I always correct them*). Reliability scores for metacognition followed the sub-constructs of awareness ($\alpha = 0.825$), cognitive strategy ($\alpha = 0.853$), planning ($\alpha = 0.842$), and self-checking ($\alpha = 0.828$). These overall reliability values were acceptable ($\alpha > 0.70$) (Tavakol and Dennick, 2011). The internal consistency of the metacognitive inventory was high, with composite reliability values (CR) ranging from 0.775 to 0.925 (>0.6). The value of the Average Variance Extracted (AVE) ranged from 0.500 to 0.526 (>0.5), indicating good discriminant validity. The square roots of all AVE values were higher than the associations suggested among them or to the left of them, underlining the discriminant validity of the metacognition scale. These values were consistent with what researchers proposed (Fornell and Larcker, 1981; Hair et al., 2010; Nunnally and Bernstein, 1994), which were satisfactory.

Strategy of data analyses. In the first analysis, we used descriptive statistics for all sub-constructs with missing data, outliers (box-plots), means, standard deviations, skewness, and kurtosis. At the same time, the relationships between latent variables were calculated using Pearson correlations to determine multicollinearity. According to Kline (2005), the relationship between the latent variables should be less than 0.900 for the observed variables to be free from multicollinearity. For the cut-off value of univariate normality, we used skewness (±2.0) (Tabachnick and Fidell, 2013) and kurtosis (±8.0) (Kline, 2005) in this paper. Then, SEM (AMOS version 18.0) was used to evaluate the hypothesized model. First, we calculated a measurement model (Confirmatory Factor Analyzes—CFA) for each variable to test whether or not the dimensional structures of the instruments could be confirmed for the sample in the present study. For the construct of metacognition, we assessed awareness models, cognitive strategy, planning, and self-checking sequentially. The following measurement model assessed two-dimensional modeling competence (horizontal and vertical mathematization). Next, we set up the hypothetical model to test the effect of the sub-dimensions of metacognition on mathematical modeling (horizontal and vertical mathematization). Model fit was assessed using the standardized root mean residual (SRMR) (<0.080), chi-square values ($P > 0.05$),

Table 1 Descriptive outputs.

Constructs	Sub-constructs	1	2	3	4	5	6
Metacogntion	1. Awareness	1	0.677	0.583	0.593	0.616	0.476
	2. Cognitive strategy		1	0.633	0.567	0.645	0.522
	3. Planning			1	0.634	0.634	0.500
	4. Self-checking				1	0.617	0.530
Modeling competency	5. Horizontal mathematization					1	0.342
	6. Vertical mathematization						1
Skewness		−0.133	−0.658	−0.124	−0.154	0.095	0.195
Kurtosis		0.842	2.343	0.087	0.106	0.032	−0.670
Mean		3.940	3.737	3.951	3.910	0.914	0.848
Standard deviation		0.552	0.668	0.584	0.637	0.331	0.523

comparative fit index (CFI) (>0.950), Tucker-Lewis index (TLI) (>0.950), the root mean square error of approximation (RMSEA) (<0.080) (Bandalos and Finney, 2018; Dash and Paul, 2021), and the goodness-of-fit index (>0.900) (Dash and Paul, 2021). SRMR was determined by taking the average of the residuals from the comparison of the observed and implied matrices (Bandalos and Finney, 2018). The chi-square test assessed the discrepancy between the observed sample data and the covariance matrices within the model. CFI and TLI compare the goodness of fit of a model to that of a null or independent model. Finally, to assess the discriminant validity, reliability, and convergent validity of the measures, we used the composite reliability (CR) (>0.60), Cronbach's alpha values (0.60–0.70), and average variance extracted (AVE) (>0.50).

Table 2 Examination of the measurement model.

Goodness-of-fit	Measurement standard	Results	
		Metacognition	Modeling competency
χ^2	$P > 0.05$	325.454	261.077
χ^2/df	<5.00	1.984	1.305
RMSEA	<0.080	0.043	0.024
SRMR	<0.080	0.036	0.041
CFI	>0.950	0.965	0.975
GFI	>0.900	0.955	0.958
TLI	>0.950	0.959	0.971

Results

Descriptive results. Table 1 shows the descriptive results and correlation matrix for the sub-construct of metacognition (awareness, cognitive strategy, planning, and self-checking) and the sub-construct of modeling competency (horizontal and vertical mathematization).

As indicated in Table 1, the highest relationship was between awareness and cognitive strategy ($r = 0.677$), while horizontal and vertical mathematization ($r = 0.342$) were the lowest correlated. Again, the students' awareness, cognitive strategy, planning, and self-checking were moderate ($M = 3.940$, $M = 3.737$, $M = 3.951$, $M = 3.910$, respectively). The skewness score ranged between −0.658 and −0.124 (± 2.0), while the kurtosis values ranged between 0.087 and 2.343 (± 8.0). The outputs indicated that no values exceeded the cut-off score for all of the four sub-constructs (Kline, 2005; Tabachnick and Fidell, 2013), which was normally distributed. At the same time, the students' horizontal and vertical mathematization were also moderate ($M = 0.914$, $M = 0.848$, respectively). The skewness score ranged between 0.095 and 0.195 (± 2.0), while the kurtosis scores ranged between −0.670 and 0.032 (± 8.0). The results showed that no scores exceeded the cut-off score for the two sub-constructs (Kline, 2005; Tabachnick and Fidell, 2013), which was normally distributed.

Measurement models. The measurement model was employed to confirm that observed variables reflected unobserved variables before evaluating the hypothetical structural model. We employed CFA to measure the fitness of the latent variables of metacognition (20 indicators) and mathematical modeling competency (22 indicators). The outputs of maximum likelihood estimation revealed that the measurement model of metacognition for the four sub-constructs indicated an acceptable match; $\chi^2 = 325.454$, $\chi^2/df = 1.984$, RMSEA = 0.043, SRMR = 0.036, CFI = 0.965, GFI = 0.955, TLI = 0.959 (Table 2). Moreover, the measurement model of mathematical modeling competency also revealed that two sub-constructs indicated an adequate fit of the model to the data; $\chi^2 = 261.077$, $\chi^2/df = 1.305$, RMSEA = 0.024, SRMR = 0.041, CFI = 0.975, GFI = 0.958, TLI = 0.971. Despite the significance of the chi-square result, χ^2/df, RMSEA, SRMR, CFI, GFI, and TLI recommended that the a priori model had an adequate factor structure.

Factor loading and coefficient of SEM regression are shown in Table 3. All factor loadings from sub-constructs of horizontal mathematization (around 0.617–0.837), vertical mathematization (from 0.660 to 0.703), awareness (around 0.662–0.738), cognitive strategy (from 0.770 to 0.758), planning (around 0.660–0.757) and self-checking (from 0.662 to 0.760), were significant. Each item within every sub-construct exhibited statistically significant factor loadings ($P < 0.001$), affirming the correlation among items for each sub-construct. The standardized estimate for factor loading indicated that all items had factor loadings greater than 0.50, which surpassed the desired criteria (Hair et al., 2010).

Testing the hypothesized models. Similar to the examining measurement model, some cut-off scores were also applied for each measurement to evaluate the hypothesized model; $\chi^2/df <$ 5.00, RMSEA < 0.080, SRMR < 0.080, CFI > 0.950, GFI > 0.900, TLI > 0.950. The results of SEM indicated a highly satisfactory fit to data, $\chi^2 = 1163.570$, $\chi^2/df = 1.460$, RMSEA = 0.029, SRMR = 0.043, CFI = 0.950, GFI = 0.908, TLI = 0.950 (see Fig. 2). The hypothesized model shown in Fig. 2 was the final structural model that indicated the relationship between the sub-construct of metacognition and mathematical modeling competency. The parameter estimates for whole structural paths in the hypothesized model were statistically significant.

Next, Table 4 shows detailed statistics on the final model (e.g., standardized estimate, unstandardized estimate, standard errors, CR, and P value).

Table 3 Factor loadings of variables.

Construct	Sub-construct	Item	Factor loading	P
Modeling competency	Horizontal mathematization	Q1	0.700	***
		Q2	0.837	***
		Q3	0.632	***
		Q4	0.751	***
		Q5	0.712	***
		Q6	0.700	***
		Q7	0.623	***
		Q8	0.700	***
		Q9	0.700	***
		Q10	0.744	***
		Q11	0.637	***
		Q12	0.636	***
		Q13	0.627	***
		Q14	0.617	***
		Q15	0.678	***
		Q16	0.708	***
		Q17	0.700	***
		Q18	0.622	***
	Vertical mathematization	Q19	0.700	***
		Q20	0.660	***
		Q21	0.670	***
		Q22	0.703	***
Metacognition	Awareness	A1	0.700	***
		A5	0.662	***
		A11	0.738	***
		A16	0.684	***
		A20	0.700	***
	Cognitive strategy	C2	0.714	***
		C7	0.717	***
		C10	0.775	***
		C15	0.758	***
		C17	0.700	***
	Planning	P3	0.660	***
		P4	0.716	***
		P8	0.731	***
		P13	0.757	***
		P18	0.729	***
	Self-checking	S6	0.760	***
		S9	0.638	***
		S14	0.735	***
		S19	0.662	***
		S12	0.715	***

***Significant.

As appeared in Table 4, the direct path coefficient was significant: (a) cognitive strategy → horizontal mathematization [$\beta = 0.26$, $P < 0.05$, $t = 2.535$], (b) planning → horizontal mathematization [$\beta = 0.23$, $P < 0.05$, $t = 2.369$], (c) self-checking → horizontal mathematization [$\beta = 0.23$, $P < 0.05$, $t = 2.470$]. The hypothesis was fully accepted. Students who used cognitive strategy, planning, and self-checking accomplished well in horizontal mathematization. Conversely, the direct path coefficient of awareness to horizontal mathematization was insignificant [$\beta = 0.17$, $P > 0.05$, $t = 1.685$]. Thus, the hypothesis was not fully supported. It implied that awareness alone might not strongly predict success in horizontal mathematization. At the same time, the direct path coefficient was not significant: (a) cognitive strategy → vertical mathematization [$\beta = 0.24$, $P > 0.05$, $t = 1.763$], (b) planning → vertical mathematization [$\beta = 0.15$, $P > 0.05$, $t = 1.180$], (c) awareness → vertical mathematization [$\beta = 0.08$, $P > 0.05$, $t = 0.635$]. It showed that awareness, cognitive strategy, and planning alone may not strongly predict success in vertical mathematization. The direct path coefficient of self-checking → vertical mathematization was significant [$\beta = 0.27$, $P < 0.05$, $t = 2.138$]. Students who used self-checking accomplished well in vertical mathematization. In conclusion, cognitive strategy (26%), planning (23%), and self-checking (23%) accounted for a variance for horizontal mathematization; at the same time, self-checking (27%) accounted for a variance for vertical mathematization.

Discussion

Integrating mathematical modeling across subject areas can give students a more meaningful and context-rich understanding of mathematics. Numerous studies have shown that many students find mathematical modeling difficult and complex (Anhalt et al., 2018; Corum and Garofalo, 2019; Czocher, 2017). For example, some students have difficulty translating real-world problems into mathematical terms, while others have difficulty finding appropriate mathematical models to represent complex systems and phenomena. This study aimed to examine whether the different sub-dimensions of metacognition could be used to predict a student's level of competency in modeling.

We found no significant or positive relationship between awareness and horizontal or vertical mathematization. Despite numerous studies that do not support the finding of a significant and positive relationship between these variables (Kreibich et al., 2022; Sevgi and Karakaya, 2020; Toraman et al., 2020), previous research has primarily focused on metacognitive awareness rather than the sub-domain of awareness within metacognition. Indeed, much of the research in mathematics education has focused on problem-solving and not specifically on the context of mathematical modeling. This focus on problem-solving has led to valuable insights into how students learn, think, and apply mathematical concepts. However, certain aspects of mathematical modeling may have been less explored or understood in the process. One possible explanation could be insufficient mathematical knowledge in mathematical modeling. Leong (2014) indicated that incorporating mathematical modeling into the curriculum may face challenges, including teacher readiness, time constraints, and educator dispositions. The extent of a student's mathematical understanding can influence the connection between awareness and horizontal or vertical mathematization. Students who do not have the requisite mathematical foundations may have difficulty making connections or applying problem-solving techniques, regardless of their level of awareness. For example, increased awareness can help students identify relevant information, recognize patterns and relationships, develop appropriate assumptions, select mathematical tools, and reflect on their modeling process.

Our results show a positive and significant correlation between cognitive strategy and horizontal mathematization; however, no significant relationship was found between cognitive strategy and vertical mathematization. This result confirms previous research in this area (Hidayat et al., 2020, 2022; Krüger et al., 2020). This observation can be attributed to the complexity of the tasks. Horizontal mathematization involves translating real-world problems into mathematical representations, whereas vertical mathematization involves working within the domain of mathematics to solve problems. Cognitive strategies, such as organizing information, recognizing patterns, and selecting appropriate tools, may be more applicable to horizontal mathematization. This result is consistent with Krüger et al.'s (2020) view that using cognitive strategies provides direction in the formulation phase of the modeling process. Conversely, in vertical mathematization, tasks may be more complex or abstract and require higher mathematical knowledge or skills. Vertical mathematization involves going deeper into the mathematical domain, working with more

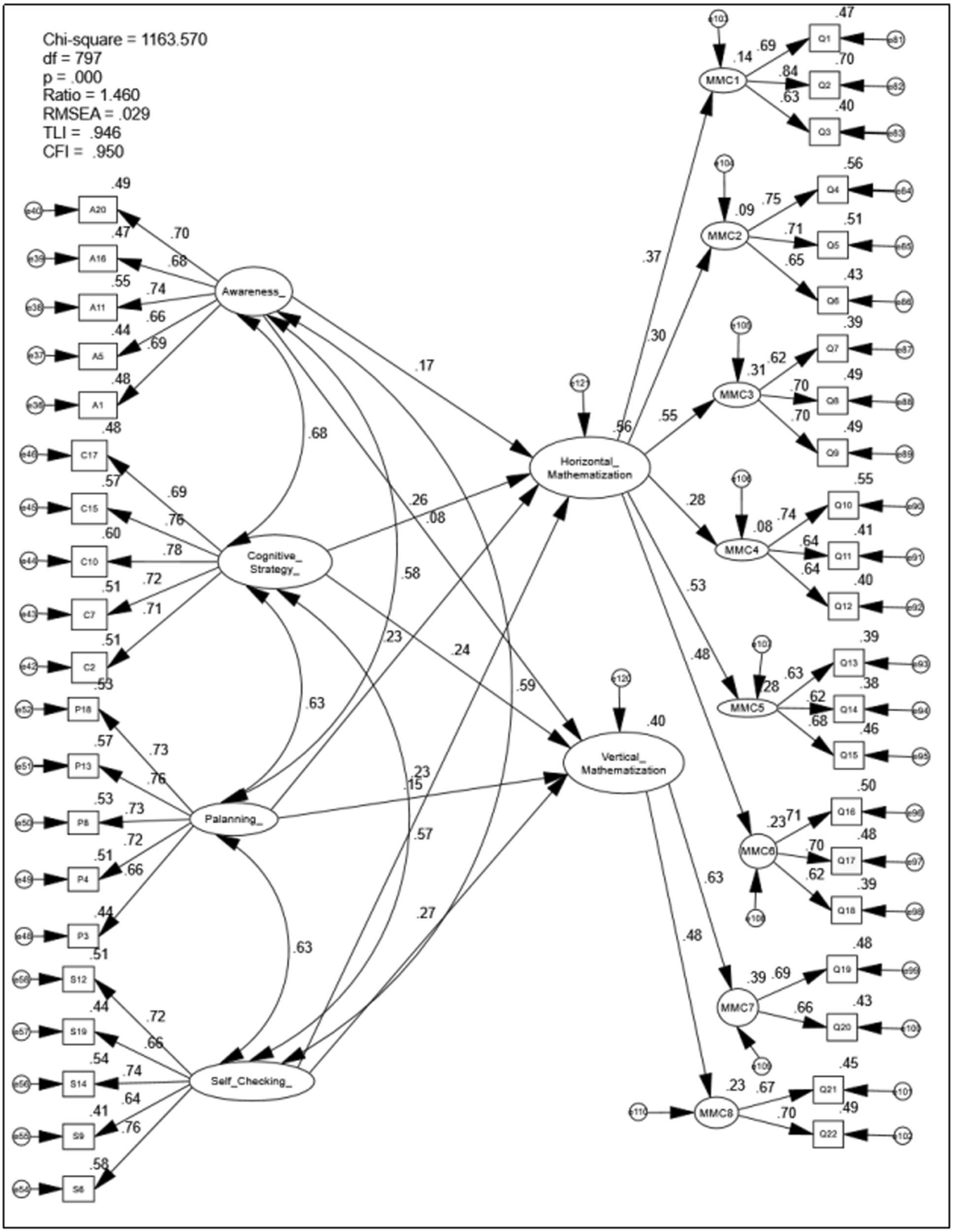

Fig. 2 The final model.

abstract concepts, and using advanced problem-solving techniques. Cognitive strategies typically focus on organizing, planning, and selecting tools that may not be as influential in this more abstract and complex domain. Consequently, cognitive strategies alone may not be sufficient to influence vertical mathematization. Another possible explanation is that students' different cognitive styles may lead to different approaches to mathematization processes. Students with different cognitive styles may lead different approaches to mathematization processes (Mariani and Hendikawati, 2017).

Table 4 Path analysis.

Hypothesis	Unstandardized estimate	Standardized estimate	Std. error	*t* value	*P*	Decision
Awareness → Horizontal mathematization	0.094	0.170	0.056	1.685	0.092	Not supported
Cognitive strategy → Horizontal mathematization	0.120	0.260	0.048	2.535	0.011	Supported
Planning → Horizontal mathematization	0.122	0.230	0.052	2.369	0.018	Supported
Self-checking → Horizontal mathematization	0.110	0.230	0.044	2.470	0.014	Supported
Awareness → Vertical mathematization	0.040	0.080	0.064	0.635	0.526	Not supported
Cognitive strategy → Vertical mathematization	0.097	0.240	0.055	1.763	0.078	Not supported
Planning → Vertical mathematization	0.069	0.150	0.059	1.180	0.238	Not supported
Self-checking → Vertical mathematization	0.113	0.270	0.053	2.138	0.033	Supported

This research's results indicate a significant and positive relationship between planning and horizontal mathematization, but no significant correlation was found between planning and vertical mathematization. This result is consistent with previous research (García et al., 2019; Herawaty et al., 2018; Zhang et al., 2019). In a horizontal mathematization context, verbalization can potentially explain this observation. Zhang et al. (2019) indicated that students can improve their problem-solving skills through planning strategies such as verbalization. Verbalization, i.e., talking about the problem and their thought processes, can also help students clarify their thinking and identify possible errors or inconsistencies in their reasoning. By breaking down complex problems into smaller, more manageable steps, students can more easily understand the problem and develop an action plan for solving it. In horizontal mathematization, students must be able to analyze the problem, identify the most important variables and relationships, and develop a plan to solve the problem using mathematical concepts and procedures. However, the sub-domain of planning is not used effectively in vertical mathematization. Vertical mathematization requires students to engage in a more analytical and abstract form of thinking, which can be more challenging than the more concrete and tangible aspects of horizontal mathematics. In addition, vertical mathematization often involves multiple mathematical concepts and procedures, making it more challenging to plan a clear and effective problem-solving strategy. Students may rely on trial-and-error methods or intuitive problem-solving approaches rather than explicit planning.

Our study shows a significant positive correlation between self-checking and horizontal and vertical mathematization. This result is consistent with previous studies conducted on this topic, such as those by Akman and Alagöz (2018), García et al. (2019), and Herawaty et al. (2018). This consistency of results between studies highlights the importance of self-checking or monitoring in mathematical modeling. One possible explanation for this consistent finding is that self-checking is beneficial for students to identify errors, ensure accuracy, and build confidence in their mathematical abilities. Using self-checking techniques, students monitor their understanding and advancement as they work through the problem. This monitoring can help them identify errors or misunderstandings early on and correct their thought processes or methods accordingly. Self-checking can also help students stay organized and focused as they solve the problem, reducing the chance of making mistakes or overlooking important details. For example, modelers correctly identified the relevant variables and relationships in the problem. Similarly, monitoring strategies can improve vertical mathematization by helping students stay organized and focused, reflecting on their problem-solving approaches, and interpreting the outcomes of their solutions. For example, monitoring or self-checking can help students interpret the results of their problem-solving efforts in the context of the original problem. By reflecting on the meaning of the solution and its relation to the real world, students can develop a deeper understanding of mathematical concepts and their applications. In addition, monitoring can help students stay organized and focused as they work through a problem, reducing the likelihood of making mistakes or missing important details. Research has shown that the sub-dimension of monitoring can predict student engagement in classroom discussions (Akman and Alagöz, 2018).

Conclusion

Mathematical modeling involves applying mathematical concepts and techniques to real-world situations and requires students to think critically, creatively, and systematically about problems. Students need opportunities to engage in various tasks that require applying their mathematical knowledge to real-world situations and sufficient time to gain experience and develop their skills. Metacognition plays an important role in mathematical modeling by helping students become more aware of their thinking processes, monitor their understanding, and decide when to seek help or additional support. According to this research, awareness alone did not significantly impact horizontal mathematization. However, using cognitive techniques, making intelligent plans, and self-checking significantly improved horizontal mathematization. To improve learners' horizontal mathematics skills, it is important to motivate them to use proper cognitive methods, acquire efficient planning techniques, and develop the habit of self-checking. In addition, the results pave the way for further research on the exact cognitive strategies, planning methods, and self-checking procedures that support effective horizontal mathematization. By analyzing how these variables interact and influence student performance, insights can be gained into instructional strategies and interventions that support successful mathematical modeling. Finally, these discoveries improve our understanding of the intricate connection between metacognition and mathematical modeling. Awareness may not directly affect horizontal mathematization, but cognitive techniques, planning, and self-checking are critical. The unique processes and techniques associated with different types of mathematical modeling must also be considered, as demonstrated by the differential effects on vertical mathematization. These findings extend our theoretical understanding of the relationship between mastery of mathematical modeling, metacognitive processes, and specific cognitive skills.

Limitations and suggestions

It is common for research studies to have limitations, and the current study is no exception. Acknowledging and considering the study's limitations in future research is essential. Firstly, some hypotheses are fully supported by the research findings, while others are not. It is possible that other factors, such as students' prior mathematical knowledge and experience, their motivation and engagement in mathematical modeling, and the quality of instruction, play a more important role in promoting horizontal and vertical mathematization. Further research is needed to fully understand the complex interplay of factors contributing to horizontal and vertical mathematization and to identify effective strategies for promoting mathematization in students. Secondly, although the current study found correlations among variables, it is important to note that correlational studies cannot prove causality. Future research may therefore benefit from using experimental designs or other methods to establish causal relationships among variables. These methods may involve interventions or manipulations designed to directly change the independent variable and observe its effects on the dependent variable. Such methods allow researchers to understand the causal relationships between variables better and draw more meaningful conclusions about the effects of various factors on the outcome of interest. Finally, a potential limitation of the current study is that it relied on self-reported measures of variables that could be susceptible to bias or error. Future research could benefit from using objective measurements or multiple data sources to increase the validity of the results. Objective measurements may include direct observation or physiological measurements, providing more accurate and reliable data. In addition, using multiple data sources can contribute to a more comprehensive understanding of the phenomenon under study, as different data sources may capture different aspects of the measured construct. Using such methods, researchers can increase the validity and reliability of their findings and draw more meaningful conclusions about the relationship between different variables.

Data availability

All relevant data can be found in the manuscript and its accompanying supplementary files.

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Author contributions

The conception or design of the work: RH. The acquisition, analysis, or interpretation of the data for the work; RH and Hermandra. Drafting the work or revising it critically for important intellectual content; RH and STDY.

Competing interests

The authors declare no competing interests.

Ethical approval

The study had permission from the Department of Investment and Integrated One Stop Services, Indonesia with number 503/DPMPTSP/NON IZIN-RISET/8323.

Informed consent

All selected samples were given a written informed consent letter. After receiving confirmation from the researcher regarding complete confidentiality and the explicit clarification that their responses would be used exclusively for academic objectives, all 538 participants voluntarily participated in the study.

Additional information

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