# Hasty basics

*Steven H. Strogatz*

**Introduction to Nonlinear Science.** By G. Nicolis. *Cambridge University Press: 1995. Pp. 254. £35, $59.95 (hbk); £13.95, $24.95 (pbk).*

NONLINEAR dynamics was a hot subject in the early 1980s. I remember the excitement and frustration of our informal seminars where a few graduate students and young professors struggled to explain unfamiliar things to one another, the blind leading the blind. We prayed for a lucid textbook.

Many books have been written since then, some mathematically rigorous, others geared more towards the physical sciences. But in the preface to his graduate text *Introduction to Nonlinear Science*, G. Nicolis argues that most of these books take too narrow a view of nonlinear dynamics — they ignore spatially extended systems as well as the statistical aspects of the subject. Nicolis tries admirably to present a coherent account of nonlinear science in a way that does justice to all its facets. Unfortunately he has not quite succeeded. In his eagerness to cover topics long neglected, he skips too many of the fundamentals.

Two examples recur throughout: thermal convection in a layer of fluid heated from below, and reaction–diffusion models of chemical oscillations and patterns. After deriving the governing differential equations, Nicolis develops tools to analyse them. First come the techniques for low-dimensional systems: phase space, linear stability analysis and local bifurcation theory. The treatment is perfunctory, except for the derivation of normal forms, handled nicely using the method of multiple scales and solvability conditions.

The same approach reappears in the more complicated setting of spatially extended systems, the strongest part of the book. Here Nicolis derives the complex Ginzburg–Landau equation and other universal models for pattern formation. He also gives clear explanations for Turing patterns in chemical systems and the various instabilities in the convection problem, for both small and large aspect ratios. A final chapter on chaos again lapses into superficiality when discussing standard topics (period doubling, circle maps and intermittency), then recovers in its treatment of ergodicity, mixing and other probabilistic notions.

The book is aimed at graduate students, but, given its hasty treatment of the basics, I would recommend it only as a supplement, rather than as a primary text. □

*Steven H. Strogatz is in the Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York 14853, USA.*

# Glimpses of fundamental harmony

*S. S. Schweber*

**The Quantum Theory of Fields. Volume 1: Foundations.** By Steven Weinberg. *Cambridge University Press: 1995. Pp. 609. £35, $49.95.*

ROBERT Oppenheimer once pointed out that all great scientific advances have two traits. On the one hand, they provide know-how, an enrichment of technique, enabling us to do what we could not do before, or to do it better; and, on the other hand, they contribute knowledge, an answer and reformulation of questions that have long excited man's curiosity, something to contemplate, "a glimpse of harmony and order, a thing of beauty".

The quantum theory of fields is no exception. It allows precise calculation of the scattering processes arising from the interaction of the electroweak and strong forces (up to energies of the order of $10^{12}$ electronvolts) and the theoretical calculations agree with empirical data to a remarkable degree. And the theory has given us a glimpse of the harmony and order of the subnuclear world.

Quantum field theory was developed during the late 1920s as a general framework to describe in quantum mechanical terms the interaction of charged particles with the electromagnetic field. During the 1930s, the framework was extended by Enrico Fermi to model beta-decay and by Hideki Yukawa to explain nuclear forces. All these field theories have two features in common: they synthesize quantum mechanics and special relativity, and the interaction between the fields is at a single point in space-time. Local quantum field theories are flawed, however: their perturbative solutions are divergent, the infinities encountered resulting from the locality of the interaction.

After the Second World War, stimulated by the reliable and precise measurements by Willis Lamb and by Isidor Rabi of the ground-state structure of hydrogen, renormalization theory was formulated to circumvent the divergence difficulties with higher-order calculations in quantum electrodynamics. Freeman Dyson showed that a 'renormalization' of the charge and mass parameters (and a rescaling of the field operators) of the Lagrangian function that describes the theory could absorb all the divergences in quantum electrondynamics. More generally, Dyson demonstrated that only certain kinds of quantum field theories can absorb *all* the infinities by a redefinition of a *finite* number of parameters. He called such theories 'renormalizable'. Renormalizability immediately became a criterion for theory selection and played an important part in the development of the standard model of the electroweak and strong interactions.

During the past 20 years, our understanding of quantum field theories and of renormalization has changed dramatically. The work of Kenneth Wilson and Steven Weinberg has given us a more limited view: all existing theoretical representations of phenomena are only partial descriptions that depend on the energy at which the interactions are analysed. Successful quantum field theories are in fact low-energy approximations to a more fundamental theory that may or may not be a field theory. Weinberg and others have shown that the reason quantum field theory describes physics at accessible energies "is that any relativistic quantum theory will at sufficiently low energies look like a quantum field theory".

Weinberg, one of the chief architects of the standard model and a major contributor to the development of quantum field theory since the 1960s, provides an impressively lucid and thorough presentation of the subject from this modern viewpoint. He believes that physics describes observable phenomena and that all physical measurements and interactions can be considered as scattering processes; and in this context he shows that quantum field theory is essentially the only way of reconciling the principles of quantum mechanics with those of the special theory of relativity. The first volume of *The Quantum Theory of Fields* covers the foundations of quantum field theory and quantum electrodynamics, whereas the second volume, due to be published next year, will deal with the important advances of the past two decades, such as non-Abelian gauge theories, broken symmetries, the renormalization group and anomalies.

Weinberg manages to present difficult topics with richness of meaning and marvellous clarity. Full of valuable insights, his treatise is sure to become a classic, doing for quantum field theory what Dirac's *Quantum Mechanics* did for quantum mechanics. I eagerly await the publication of the second volume. □

*S. S. Schweber is in the Department of Physics, Brandeis University, Waltham, Massachusetts 02254, USA.*