= stretch(pref(X)) on which his values prob(X) obey prob (A) + prob(not$A$) = 1, prob(A,B) = prob(A) prob (B given A).

These rules define the only general calculus for patterns containing up to three bits. Larger patterns can be built up by investigating their bits sequentially, during which the rules remain consistent as well as being demanded. Thus a generally applicable calculus can exist, proving existence and uniqueness.

Cox's stretched values are probabilities, and the product rule immediately gives Bayes' theorem, the required tool for inference.

Edwards discards axiom 1, because he has improperly started in an infinite space where he cannot recover a single element by double negation. But in my finite computer, no patterns occupy less than 1 part in 2^{billion} of the space. Negation is the complement operator, which remains reversible for any finite computer: thus $2^{\text{billion}} - (2^{\text{billion}} - 1) = 1$.

Bayesian probability calculus is rock solid and, as Edwards admits, "statistical theory and statistical practice [can thereby] be greatly simplified".

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SIR — The example of Fisher's quoted by Edwards[1] is a straw man to discredit the opposing view. Only in this type of argument would a bayesian practitioner confronted with the black mouse give a probability of the mouse being homozygous of one half when s/he knew nothing about any possible relationship between colour and genotype. In practice, a real bayesian practitioner would observe that s/he knew nothing about the genotype and so would assign a uniform prior distribution over the interval 0,1 to the unknown probability. Informally, this means that the bayesian thinks that the probability of the mouse being homozygous lies between 0 and 1, while the knowledgeable scientist believes it is one-half. The seems to be a satisfactory situation, given their respective states of knowledge and belief.

The prior distribution used is subjective, as stated by Howson and Urbach in their Commentary[2]; however most statistical procedures contain subjective or arbitrary elements (the significance levels for rejecting hypotheses, for example). Here at least the assumptions have to be stated explicitly. In practice, of course, this is not a real difficulty as posterior distributions generally show only weak dependence on the form of the prior for most reasonable prior distributions when a useful amount of observational or experimental data is available.

It is possible that some non-bayesians (anti-bayesians?) have a conceptual difficulty with the idea of the probability of a probability, which could lead to the above approach being ignored. Fisher's misinterpretation of what to do about ignorance of the values of probabilities in a bayesian argument, is at the root of many arguments against the use of bayesian methods in expert systems, and as we see, this argument is invalid.

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Quasar redshifts

SIR — Evidence in support of the cosmological origin of quasar redshifts, which comes mainly from the absorption features in quasar spectra does not seem to have settled the "quasar controversy"[1], as evident from the 'Hypothesis' by Arp *et al.*[2], who argue that the quasars lie at much smaller distance than their redshift distance, and thus that quasar redshifts are largely intrinsic in origin and not cosmological. However, recent observations on gravitational lensing of quasar images provide clear-cut evidence that quasars lie at their redshift distances.

Einstein's theory of general relativity predicts that a galaxy with a radially symmetric surface density that happens to lie on or very near the line-of-sight to a distant quasar, forms in the sky a ring image[3] (Einstein ring) of the quasar with an angular diameter[4] $\Delta\theta \approx 4\pi(D_{LS}/D_{OS})(v_{cir}/c)^2$, where D_{OS} and D_{LS} are the angular distances from the observer to the source (quasar) and from the lens (galaxy) to the source, respectively, and v_{cir} is the rotational velocity in the lensing galaxy at the bending point. Such an Einstein ring, MG1654+1306, of a radio lobe of a quasar at redshift $z_s = 1.75$ was discovered by Langston *et al.*[5,6]. It is formed by a bright elliptical galaxy at redshift $z_L = 0.254$ and it has an angular diameter of $\Delta\theta = 1.97'' \pm 0.04''$. If the quasar and the galaxy lie at their redshift distances then $D_{LS}/D_{OS} \approx 0.73$ and the expected angular diameter of the ring from the estimated[5] circular velocity in the lensing galaxy ($v_{cir} \approx 330 \pm 20$ km s^{-1}) is $\Delta\theta \approx 2.09'' \pm 0.27''$, in good agreement with the observed value. On the other hand, if the quasar lies in or near the lensing galaxy (that is, if $D_{LS} \leqslant 50$ kpc) then $D_{OS} \approx D_{OL}$, $D_{LS}/D_{OS} \leqslant 50$ kpc/$D_{OL} \leqslant 10^{-4}$ and $\Delta\theta$ would be less that 2×10^{-4} arcs, which vastly contradicts the observations.

When the lensing galaxy has an elliptical surface density, the Einstein ring degrades into four images that are located symmetrically along the two principal axes[7] as observed[8,9] in the case of Q2237+0305, where the lensing galaxy has a redshift $z_L = 0.0394$ and a rotational velocity of ~ 260 km s^{-1} (ref. 8), and the quasar images have redshifts $z_S = 1.695$. The predicted angular separation between opposite images (roughly the diameter of the ring) is $\Delta\theta \approx 1.85''$, in very good agreement with the ground-based observations[8,9] ($\Delta\theta = 1.75'' \pm 0.10''$), and with recent observations from the Hubble Space Telescope ($\Delta\theta = 1.78'' \pm 0.05''$). The system Q2237+0305 was highly advertised as an evidence that a high-redshift quasar lies very near the centre of a low redshift galaxy. However, this would produce an angular separation between opposite images smaller than that observed by at least three orders of magnitude.

One may also compare the observed time delay[10,11] of 410±10 days between the two images of Q0957+561 at angular distances $|\vec{\theta}_A| = 5.24'' \pm 0.5''$ and $|\vec{\theta}_B| = 1.00'' \pm 0.5''$ from the centre of the lensing galaxy at redshift $z_L \approx 0.36$ and the predicted time delay[12] $\Delta t_{A,B} \approx 4\pi(1+z_L)(|\vec{\theta}_A| - |\vec{\theta}_B|)(\sigma_{\parallel}/c)^2(D_{OL}/c)$. A recent measurement of the line-of-sight velocity dispersion in the giant galaxy gave[13] $\sigma_{\parallel} = (303 \pm 50)$ km s^{-1}. Using the best value $H_O = 67 \pm 8$ km s^{-1} M pc^{-1} for the Hubble parameter, found in a recent review[14], the predicted time delay is 416±130 days, in agreement with the observed time delay. However, if the quasar lies in or near the lens, the deflection angle, and consequently the expected time delay, would be smaller by more than four orders of magnitude than the observed values.

In all other known cases of gravitational lensing of quasar images by galaxies (and clusters of galaxies) the angular separations between the multiple images are of similar magnitude[15]. Moreover, the number of lensed quasars that are observed is that expected[4] if the quasars lie at their redshift distance.

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