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Lukash and Novikov have given an interesting survey of their analyses of some of the features that may exist in the microwave background radiation in open universes. Here, we answer some criticisms that they make of our earlier analysis[1], and point out a further unusual feature that appears in the temperature distribution of the most general flat and open cosmological models.

Lukash and Novikov claim that in ref. 1 we erroneously represented infinite wavelength density perturbations by ensembles of Bianchi type VII universes at linear order, rather than by cosmological models of Bianchi types I or V, as required. This claim is incorrect. We found, and stated, that infinite-wavelength, pressure-free density inhomogeneities could be represented by ensembles of VII models only when one of their defining parameters, $x = h^{1/2}(1-\Omega_0)^{-1/2}$, tends to infinity. In this limit, the type VII_h and VII_0 models specialize identically to the type V and I models respectively; here, Ω_0 is the density of the Universe today in units of the critical density and x or h is an arbitrary constant once Ω_0 is specified. (In the VII_0 case, the parameter x still exists when the limits $h \to 0$ and $\Omega_0 \to 1$ are taken simultaneously; see ref. 2 for details). Accordingly, all computations in ref. 1 of the effects of inhomogeneity on the background radiation were performed using the type I and V models, as Lukash and Novikov argue is necessary.

Finally, we would like to comment on one important difference between the hotspot patterns that arise in open universes of type VII and those arising in open universes of type V which were described by Lukash and Novikov and by ourselves[1]. Although the type VII models are not relevant for the description of density inhomogeneities, they can describe the effects of gravitational waves and vorticity on the temperature anisotropy of the microwave background.

The general equation used by Lukash and Novikov to describe the hotspot structure is based on the type V paradigm where the spatial curvature is isotropic, and does not completely describe the most general hotspots generated by vortical and gravitational wave distortions; these will be described by the type VII_h behaviour rather than the simpler type V case. The geodesics in open type V universes exhibit focusing of a quadrupole temperature profile into a hotspot feature but with no change in the azimuthal coordinate of the geodesics propagating from their last scattering to the observer today. However, in type VII models there arises a geodesic spiralling effect which is superimposed on the hotspot structure. This feature was first noted by Collins and Hawking[3] and its observational effects are examined in detail in ref. 2. Figure 1 shows examples of the temperature anisotropy patterns predicted in VII_0 and VII_h universes. In the VII_0 case, where $\Omega_0 = 1$, there is a pure quadrupole plus the geodesic spiralling effect, whereas in an open VII_h model with $\Omega_0 = 0.7$, we see a spiral plus quadrupole pattern focused into a spiral hotspot. The number of turns of the spiral is inversely proportional to the parameter x. In the limit $x \to \infty$, in which case the spatial curvature becomes isotropic, the VII_0 and VII_h models become those of types I and V again, the spiral effect disappears and we recover either a quadrupole or the simple hotspot structure discussed in the preceding letter and ref. 1. Note that because this spiral feature does not arise in closed universes, it may enable observers to determine whether the Universe is open or closed, no matter how close its density lies to the critical value[2].

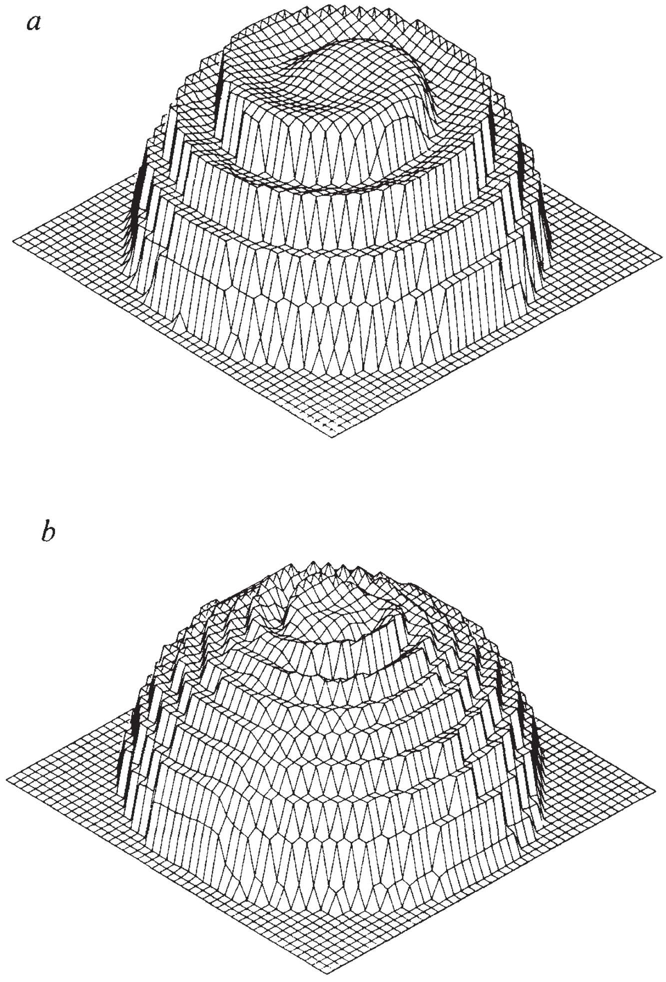

Fig. 1 Microwave background temperature profiles, $T(\theta_0, \phi_0)$, predicted in the region of observed angles $0 < \phi_0 < 2\pi$, $\pi/2 \leq \theta_0 < \pi$. The radial distance represents the magnitude of $T(\theta_0, \phi_0)$. We assume that the last scattering of the photons occurred at a redshift of 1,000, and the anisotropy amplitude is taken to be the maximum allowed by current observations. *a*, Spatially homogeneous-type VII_0 cosmological model with $x = 0.067$ and $\Omega_0 = 1$. *b*, An open spatially homogeneous cosmological model of type VII_h with $x = 0.067$ and $\Omega_0 = 0.7$. In *a*, the pattern is approximately a pure quadrupole modulated by the geodesic spiralling effect. In *b*, both the quadrupole and spiral patterns seen in *a* have been focused into a region of smaller angular scale towards the axis $\theta_0 = \pi$ by the negative spatial curvature. The result is a spiral hotspot. The spirals are left-handed in the observed angles. As $x \to \infty$ the spiral effect disappears and the hotspot in *b* degenerates into the simple variety found in the type V models, and *a* becomes the pure quadrupole of type I. For further details and discussion see ref. 2.

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