# LETTERS TO NATURE

## PHYSICAL SCIENCES

## Flying Clocks and the Sagnac Effect

IN their "flying clock" experiment[1] Hafele and Keating observed an on-Earth directional dependence of the relativistic time dilation. I have argued[2] that such a dependence is contrary to special relativity theory; the effect, however, is essentially one involving accelerated motion[3] and my neglect of this fact invalidated my argument. None the less, locally the time effects on moving clocks may be regarded as special-relativistic (neglecting the altitude effect, which is not relevant for the present discussion); and the derivation of the effects was made by Hafele with the customary $(1-v^2/c^2)^{\frac{1}{2}}$ time-rate change factor[4,5]. A seeming inconsistency then still arises in considering the clocks from the Earth reference frame: if two similar clocks are moving on equatorial paths with equal speeds relative to the Earth, one westward and one eastward, they should have equal time rates and equal kinetic energies in the Earth system. Evidently, they do have equal energies, but not equal time rates. But if the Sagnac effect is taken into account in the synchronization of clocks in the Earth frame the contradiction disappears; one finds, rather, a further exemplar of consistency in the theory of relativity

With omission of the altitude term, the equation[4] which was confirmed by the Hafele–Keating observation is

$$\Delta t' \simeq (1 - v^2/2c^2 - vR\Omega/c^2)\Delta t \qquad (1)$$

$\Delta t$ is time interval for a rest Earth clock and $\Delta t'$ is for a clock moving on the equator with speed $v$, (+) for eastward, (−) for westward motion; $\Omega$ is the Earth's axial rotation velocity, and $R$ its radius (to be taken with a cos $\lambda$ factor for east–west motion at latitude $\lambda$).

The Sagnac effect[6–8] requires that the time for light to be reflected around a closed path in a system rotating with angular velocity $\Omega$ will be longer (shorter) by $2A\Omega/c^2$ than when $\Omega=0$, if the sense of the path is the same as (opposite to) that of $\Omega$; $A$ is the area enclosed by the light path. Synchronizing rest clocks along the equator, using the Einstein procedure[9], requires that $t_2-t_1=L/c$, where $t_2$ and $t_1$ are reception and emission times, respectively, for a light signal sent the Earth distance $L$ from clock "1" to clock "2". For signals directed eastward, additional time $2\pi R^2\Omega/c^2$ will be required for traversal around the Earth, compared with the time that would be given by clocks in a hypothetical non-rotating Earth system S. Hence the equatorial clocks fall behind, compared with S clocks, as we progress eastward; the Earth observer, using the $\Delta t = L/c$ criterion, does not take into account the increase in light path that results from the Earth's axial rotation. (The equatorial rest clocks also are presumably running slow by a uniform $\Omega^2R^2/2c^2$ factor, compared with the S clocks.)

In moving a distance $\Delta x = v\Delta t$ the eastward flying clock should lose $(v^2/2c^2 + vR\Omega/c^2)\Delta t$ s compared with Earth rest clocks. But if these are synchronized by the Einstein procedure, an eastward displacement of $\Delta x$ gives a fraction $\Delta x/2\pi R$ of the $2\Omega A/c^2$ Sagnac loss, which is, using $A=\pi R^2$, a loss of $(\Omega R/c^2)\Delta x$. This decrease is the same as the loss, $(vR\Omega/c^2)\Delta t$, $\Delta t = \Delta x/v$, that is prescribed by equation (1). Hence, the Earth observer will not see a direction time change for the moving clock, but only the kinetic $(v^2/2c^2)\Delta t$ loss. Similarly for a clock moving to the west: the $(+vR\Omega/c^2)\Delta t$ gain will be compensated by the increase that the Sagnac effect gives to the synchronized clocks. For the Earth observer, then, each flying clock loses time only by the $-v^2/2c^2$ factor.

But at some point there must be a discontinuity of $2A\Omega/c^2$ in the equatorial clock system, because the Sagnac effect puts that loss (gain) into an eastward (westward) synchronization around the equator. The $(vR\Omega/c^2)\Delta t$ term of equation (1), with $\Delta t = 2\pi R/v$ for a circumnavigation of the Earth, also gives the time difference $2\pi\Omega R^2/c^2 = 2\Omega A/c^2$, in exact agreement with the synchronization discontinuity. Also, we see from the last calculation that the difference is independent of the speed $v$ at which the clock is moved. Hafele and Keating did confirm a $\pm 2A\Omega/c^2$ time difference ($\sim$200 ns, $\lambda=0$) for clocks carried around the Earth, compared with a rest clock.

For equatorial clocks set by meridian transit of star (in effect, synchronized with S coordinate clocks), the east–west time effect will appear. With Einstein synchronized clocks there would be no directional effects along a given line of latitude, but there would be what we might call the Hafele–Keating discontinuity, of magnitude $2A\Omega/c^2$ and, like the International Dateline, required for single valued time measure. Because $A$ varies with $\cos^2\lambda$, a continuous change of time would then also be required along each line of longitude. There would be no physical basis in north–south signal propagation for this last time variation. But with any idealized spherical Earth time-mesh there would be a north–south signal insensitive discrepancy; a $(1-\Omega^2R^2\cos^2\lambda/c^2)^{\frac{1}{2}}$ clock-rate slowing factor that is uniform at any given $\lambda$. Seemingly, the asymmetry in clock synchronization that comes with the Earth's axial rotation cannot be escaped.

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## Possible Marginal Fracture Ridge south of South Africa

A NOTABLE feature of the continental margin south-east of South Africa is its steep slope. Off the west coast of South Africa, north of Cape Town, a thick wedge of sediments has given the slope a gentle gradient of about 1.5° (ref. 1), while southwards from Cape Town the gradient is about 5°. But when the slope turns abruptly (at the tip of Agulhas Bank,