LETTERS TO NATURE

PHYSICAL SCIENCES

Ehrenfest's Paradox

ATWATER's remarks[1,2] on the relativistic rotating disk problem (Ehrenfest's paradox[3]) have elicited considerable response[4-6] of a somewhat peripheral nature. Atwater concludes that recourse to experiment is highly desirable. Arzelies[7], while questioning the existence of relativistic constraints, also agrees with this view. The purpose of this communication is to point out what one might expect to observe as a disk rotates and to develop the magnitude in a practical case.

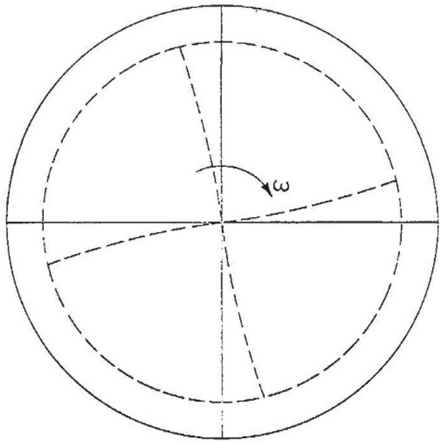

Fig. 1 A schematic representation of the rotating disk.

Suppose we have a disk of radius a rotating clockwise with angular velocity ω about an axis through its centre perpendicular to its plane. Take a radial element of the disk at distance r from the centre and consider a small section Δr along r. Then after one revolution, this section must have rotated counterclockwise with respect to its direction along r in consequence of the Thomas precession[8].

It is easy to see that this angle (in radians) is given by

$$\delta\theta = \pi \frac{\omega^2 r^2}{c^2} \qquad (1)$$

per revolution. Integrating along the whole element, we find it deflected counterclockwise by an angle

$$\theta = \frac{\pi}{3} \frac{\omega^2 a^2}{c^2} \qquad (2)$$

per revolution.

Thus the appearance of the disk to a fixed observer will be as shown by the dotted lines in Fig. 1. It will be noted that the disk is in torsion with consequent reduction of both radius and circumference. Thus the premises leading to the Ehrenfest paradox need not exist. This description should not be any more surprising than the well observed time dilation. A rotating disk is clock-like in character with the Thomas precession giving an accumulating measure of the number of revolutions. Further, the effect should be independent of the disk material, just as an equivalent clock is independent of its construction.

The relativistic constraints just introduced do, moreover, imply physical consequences for rotating bodies such as the Earth, pulsars, and so on, but this is considered elsewhere (D. H. W. and J. W. Kern, to be published). I wish to make clear that a practical laboratory experiment is entirely possible.

Take, for example, a disk 10 cm in diameter rotating 1,000 r.p.s. Then equation (2) yields $\theta \sim 10^{-12}$ per revolution. For a period of 30 days $\theta \sim 0.16°$. This should be well within currently available means of measurement when the position of a point on the circumference is compared with a point near the axis.

This measurement might be conveniently carried out, for example, by placing a fiducial mark on the edge of the disk with a similar mark in the stationary frame adjacent to the edge. These marks can then be compared during rotation when illuminated by a discharge lamp triggered by a sharp magnetic pulse, or other means, positioned at a radial distance b on the disk. This will modify equation (2) and it then becomes

$$\theta = \frac{\pi}{3} \frac{\omega^2}{c^2} a^2 \left(1 - \frac{b^2}{a^2}\right) \qquad (3)$$

per revolution.

With the passage of time, the mark on the disk will appear to move away from the fixed mark in accordance with equation (3) and counter to the direction of rotation.

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Geomagnetic Dynamos in a Stable Core

Higgins and Kennedy[1] have suggested that the Earth's core is in stable equilibrium. If this is so there cannot be the large scale steady motions with a radial component which have