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Appendix 1. Searches for quarks in 30 GeV proton bombardments of metal targets show no measurable production of particles of non-integral charge  $[2/3 e], [4/3 e], [5/3 e], \dots$  thus no production of  $QQ, QQQQ, \dots, Q_{3n}$ , at least the second of which should have a mass about equal to the proton mass if the  $Q$ - $p$  binding is appreciable, and thus should be produced with 30 GeV. This indicates that quarks bind only in threes and on this account it is generally hypothesized that quark strong binding forces saturate at  $n=3$  (ref. 9). Strong binding of one quark to a nucleon or nucleus apparently does not occur.

Appendix 2. The effect of adding a static negative quark to a fissionable nucleus has negligible effect on the fission threshold. This is illustrated in ref. 10, Fig. 14-3, where activation energy is plotted for spontaneous fission against  $A$  for various values of  $Z$ . These curves were computed by S. Frenkel and N. Metropolis using the Los Alamos Maniac Computer and the Bohr-Wheeler formula. For uranium ( $A=235, Z=92$ ) the activation energy is 7.6 MeV. Addition of a negative quark results in a nucleus having  $A=240, Z=91.7$  or 91.3. The activation energy for the new nucleus is seen to be 8.5 or 9.0 MeV, respectively, little changed from that for the nucleus without the quark.

Appendix 3. We compute the rate to produce fission caused by a quark in a bound orbit as follows. The Weizsacker-Williams formula describes the electromagnetic field of the quark as a system of light quanta. One of the quanta may cause fission as the bound quark orbits the uranium nucleus. The cross-section for this process is the cross-section for photo-fission multiplied by the number of equivalent quanta per pass. The total number of equivalent quanta per pass is given by,

$$(1/2) Z Z' (e^2/\hbar c) (dk/k) (4/\pi^2) (\ln(E/k) - 0.39)$$

where we shall take  $E \sim 20$  MeV,  $k \sim 10$  MeV,  $dk \sim 1$  MeV, and the bracket containing the logarithm  $\sim 0.1$ . Then the number of equivalent quanta per pass is approximately (representing the fine structure constant as  $a$ )

$$(2/17)(30 \cdot 10^{-3}) a(10^{-3})$$

where an extra factor of  $10^{-3}$  has been introduced to take into account the velocity of the quark being less than  $c$ . This is to be multiplied by the photo-fission cross-section, for example, a 10 MeV quanta, of  $\sim (10^{-24}/a)$  cm<sup>2</sup> and by the number of uranium nuclei per cm<sup>2</sup> as seen by the bound quark of  $\sim 1/(4\pi r^2) (6 \times 10^{23})$  cm<sup>2</sup>. One finds then the probability that a bound quark making one pass by the uranium nucleus will cause fission is  $\sim 10^{-5}$ . The quark orbits the uranium about  $10^{23}$  times per second and thus will cause fission in about  $10^{-18}$  s after capture. The important rate is thus the time to be captured, which is about  $10^{-13}$  s, a radiative process.

Appendix 4. The shortest length of a fission chain is estimated in the following way. We compute the time  $t$  in which a quark and a fission fragment, moving with parallel velocities, remain no further apart than the radius of an outer Bohr orbit, and compare  $t$  with the time  $T$  in which to radiate the binding energy of the quark to that orbit. Then the probability of quark fission-fragment capture  $P$  is

$$P \leq \frac{1}{2} (1 - \cos \theta) (t/T)$$

where 2 takes into account both fission fragments, and  $\theta$  is a conservative estimate of the allowable angle between quark and fission-fragment velocity vectors. Assuming a principal orbit quantum number  $n \sim 100$ , and for the fission-fragment a charge  $Z_{ff} = 6$  and mass 140 (which gives a slow fragment velocity and a maximum fission yield) then the radius of the outer orbit is  $r = 4 \times 10^{-10}$  cm, and the binding energy is 1.8 keV. We compute  $T$  from the relation

$$1/T = (4/3) (\omega^2 c^2 / \hbar c^3) r^2 = 1/3 \times 10^{-12} \text{ s}$$

We compute  $t$  from the relation

$$t \leq r / (v_{ff} - v_q) = 1.6 \times 10^{-12} \text{ s}$$

where  $v_q \leq 6 \times 10^9$  cm/s for a quark of  $\leq 1$  MeV kinetic energy and  $v_{ff} = 3.2 \times 10^9$  cm/s. (The quark causes fission as soon as it has 7 or 8 MeV binding energy to the uranium, but loses 7 or 8 MeV in so doing.) For  $t$  to have this value within 10 per cent,  $\theta \leq 35^\circ$ . Then the probability of capture of quark to fission-fragment  $P$  is

$$P \leq 8 \times 10^{-3} \text{ per fission}$$

and the number  $N$  of fissions in each chain is

$$N > 1.3 \times 10^8 \text{ fissions.}$$

For  $Q^{-2/3}, Q^{-1/3}, Q^{-5/3}, \dots$  the chains should be longer. The random walk path length  $L$  for a chain of  $N = 6 \times 10^8, L = d N$ , taking  $d$ , the distance the quark travels between each fission, as  $v_q (10^{-13} \text{ s}) = 6 \times 10^{-5}$  cm, is 0.02 cm. Thus the probability of quark loss by random walk is small.

The probability that the quark rides off bound to a fission fragment is not directly related to the probability of emission of deuterons, tritons,  $^3\text{He}$ , alphas, etc., from nuclei. The probability of emission of these is larger by orders of magnitude than can be accounted for by cascade theory, suggesting that these entities already exist inside the nucleus, instead of being formed by pick-up.

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## Elastic Constants of Magnesia, Calcia and Spinel at 16 GHz and 4.2° K

THERE is wide interest in the study of the propagation of high frequency ultrasound through single crystal media, particularly as a technique for the investigation of spin-lattice relaxation phenomena. Experiments have therefore been carried out at a frequency of 16 GHz to obtain ultrasonic pulse echoes in single crystals of magnesia, calcia and spinel.

The experimental arrangement for ultrasonic generation is similar to that used by other workers at lower frequencies (for example, ref. 1). The increase in effective ultrasonic attenuation caused by scattering due to surface imperfections and the decrease in transducer efficiency, which occur with increasing frequency, however, necessitate very careful preparation of crystal surfaces. Furthermore, because of the reactivity of calcia, it was found necessary to give a final polish to these specimens only seconds before bonding to a quartz transducer. The bond was formed of twin pack 'Araldite' which was subsequently cured. The free ends of the crystals were also protected by thin films of 'Araldite', both during the curing of the bond and the subsequent experiment.

Table 1. ELASTIC CONSTANTS AT 16 GHz AND 4.2° K

Material	Elastic constant	Experimental determination (in units of $10^{11}$ dynes/cm <sup>2</sup> )
Magnesium oxide	$C_{11}$	$31.2 \pm 0.6$
Calcium oxide	$C_{11}$	$27.6 \pm 5$
Spinel	$C_{11}$	$29.9 \pm 0.6$
	$C_{12}$	$16.5 \pm 1.2$
	$C_{44}$	$16.5 \pm 1.2$

With suitably prepared crystals it was found possible to propagate 16 GHz longitudinally polarized ultrasound along the (100) axes of magnesia and calcia and along the (100) and (110) axes of spinel. The corresponding elastic constants have been evaluated and the results are presented in Table 1, the values of  $C_{12}$  and  $C_{44}$  in the case of spinel being inferred from the Cauchy relationship<sup>2</sup>.

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## Multiple Collinear Laser-produced Sparks in Gases

THERE has been much interest in the recently reported<sup>1-4</sup> ionization and breakdown in gases induced by the focused output ( $\sim 20$  MW) of Q-switched lasers. A puzzling feature of the phenomenon is the occurrence<sup>5,6</sup> of distinct collinear regions of intense ionization along the laser beam axis near the focal point of the short focal length lenses commonly used in this work. There seems to be no satisfactory explanation of the mechanism producing these quite distinct regions. Considerations based on the development of a radiation supported shock wave<sup>7</sup> or travelling ionization waves<sup>8</sup> seem to be more appropriate to the later stages of breakdown when the local ionized regions coalesce to form a long spark which appears to be