BOOK REVIEWS

ALGEBRAICAL GENIUS

The Mathematical Papers of Sir William Rowan Hamilton
Edited by H. Halberstam and R. E. Ingram. Vol. 3: Algebra. (Cunningham Memoir, No. 15.) Pp. xxiv + 672. (London: Cambridge University Press, 1967.) 210*s*. net.

HAMILTON'S decisive contribution to algebra is his discovery of the quaternions. It is the source from which modern algebra has derived some of its most fertile ideas and methods: the freedom to discard the commutative law of multiplication and the rigorous construction of new algebraical systems from those already known, thus putting algebra on a sound logical basis. Hamilton was deeply concerned about the foundations of algebra which, in his day, were considered to be far less secure than those of geometry. The introduction of negative numbers and, to a greater degree, of complex numbers appeared to be obscure and even illogical to many of his contemporaries (and perhaps still to some of our own). In a long essay, written in 1833, he propounds the seemingly bizarre doctrine that "algebra is the science of pure time"; in more prosaic language this means that he proposed to take the positive real numbers as a starting point whence the continuum of all real numbers, positive or negative, can be derived as equivalence classes of pairs of positive numbers, equivalent pairs having the same "difference". Granting now the real numbers, he proceeds to give his celebrated construction of the complex numbers as pairs of reals, obeying the multiplication law

$$(a_1,b_1)(a_2,b_2) = (a_1a_2 - b_1b_2,\ a_1b_2 + a_2b_1)$$

From there it was a natural step to pass to an algebra of triplets endowed with a product that preserves the usual rules of algebra. But for thirteen years Hamilton was held up by the fact (as we now know) that no "division algebra" of triplets can possibly exist. Time and again, after a night of strenuous efforts, his children asked him in the morning, "Papa, can you multiply triplets?", to which he replied, with a sad shake of his head, "No, I can only add and subtract them". It was on October 16, 1843, that "a spark flashed forth" and Hamilton discovered that a satisfactory multiplication could be found for quadruples, or quaternions, as he called them, rather than for triplets. He believed that this discovery was his greatest scientific achievement and he hoped that quaternions would turn out to be a powerful tool in dealing with geometrical and physical problems. He devoted numerous articles to the applications of quaternions, some of which contain results of remarkable elegance. Looking back, more than a hundred years after Hamilton's death, one would admit that the usefulness of quaternions fell short of his expectations despite isolated successes, which persist to our own days, in various branches of mathematics. On the other hand, the theoretical ideas which led him to his discovery had a more far-reaching influence on the development of algebra than might have been anticipated at the time.

The papers on quaternions and related topics occupy about three-quarters of this volume, the remainder being taken up by Hamilton's researches on the quintic, now mainly of historical interest, and his ingenious invention of the "icosian calculus", a precursor of modern graph theory exemplified by the network of edges on an icosahedron.

Hamilton's memoirs were composed in an age in which a scholar's time and a printer's wage were considered less precious than at present, and an author was at liberty to expound his work in full detail and to adorn it with personal digressions, usually cast in impeccable prose, which gave mathematical writings a much more personal flavour than is possible today. It is interesting to learn how deeply Hamilton was influenced by John T. Graves, whose name is not widely known to mathematicians of our time.

It need scarcely be stressed that the volume has been beautifully produced by the Cambridge University Press. The editors, Professor H. Halberstam and Dr R. E. Ingram, have provided a lucid introduction and valuable addenda, which round off the monumental work of one of the great creative minds in the history of the mathematical sciences.

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PULSE COMPRESSION AND RADAR

Radar Signals
An Introduction to Theory and Application. By Charles E. Cook and Marvin Bernfeld. (Electrical Science; A Series of Monographs and Texts.) Pp. xvi + 531. (New York: Academic Press, Inc.; London: Academic Press, Inc. (London), Ltd., 1967.) 156*s*.

MODERN radar systems, although they function in accordance with the same basic principles as the primitive radars of some twenty years ago, are a million times more sensitive. The use of larger aerial systems, higher operating frequencies, more powerful transmitters and receivers of lower noise levels have all contributed to the improvement, but the most important contribution has come from the realization, during the 1950s, that in pulsed radar the range resolution and accuracy are functions of the signal bandwidth and not of the transmitted pulse width. The maximum detection range can be obtained by using the maximum energy per pulse and thus the widest pulse in accordance with the energy requirements of the system or with the available transmitter power. Nevertheless, even after this pulse width has been chosen, it is still possible to satisfy the range resolution conditions by coding the transmitted signal with wideband modulation information. The extraction at the receiving end of the information does require, however, a more complex receiving system than that normally required for a simple pulse radar. Such a complex receiving system is called a matched filter processing system, and the signal processing generally implies operations performed on the received signal in the r.f. or i.f. portions of the radar receiver.

Both the concepts and techniques involved in the pulse compression, matched filtering radar signal processing system are fully described in *Radar Signals*. The first five chapters are devoted to the broad theoretical aspects related to matched-filter techniques, with detailed discussions of the principle of stationary phase, the radar ambiguity function, and parameter estimation. The next four chapters deal with specific radar waveforms, including the linear FM waveforms and discrete coded waveforms. In the final four chapters various practical problems associated with matched filter systems are considered. These include effects of distortion on matched filter signals, the design of lumped constant and ultrasonic dispersive delay lines, and the application of microwave and optical techniques to matched filter designs.

The text is addressed primarily to the systems engineer, and although it is extremely clearly and well written and amply supplied with more than 300 illustrations, it demands of the reader a fairly high standard of mathematical knowledge and acquaintance with waveforms and wave analysis. Each chapter is adequately supplied with a list of references, varying in number from eight to thirty-eight, and totalling 268. These, together with a bibliography of sixty-nine references and detailed author and subject indexes, indicate the care which the authors