$$\frac{dE}{dt} = n_c m_0 V^2 = 3\left(\frac{3}{5}\,Gm_0\right)^{3/2} m_0 \frac{N^{7/2} r^2}{R^{9/2}} \approx \frac{1 \times 10^{11}\, N^{7/2}(r/r_0)^2}{R_{\text{pc.}}^{9/2}} \text{ erg/sec}$$

Thus if $r = r_0$, $N = 10^{10}$ and $R = 1$ pc. we find $dE/dt = 10^{46}$ ergs/sec, a factor of ten larger than the radiative losses from the strongest radio galaxies. The total available kinetic energy is:

$$E = \tfrac{1}{2} N m_0 V^2 = \frac{3}{10}\frac{Gm_0^2 N^2}{R} = 3 \times 10^{40} \frac{N^2}{R_{\text{pc.}}} \text{ ergs}$$

or with the parameters chosen 3×10^{60} ergs. The time scale of the source then becomes 10^7 years. Small changes in the adopted parameters would enable one to cover the full range of radio galaxy luminosities. If during the collisions the stars would also release 1 per cent of their nuclear energy content the available energy would be doubled.

The angular momentum difficulties in the formation of these nuclei are much less severe than in the Hoyle–Fowler objects. If we measure the angular momentum per unit mass in (km/sec)pc. then a nucleus of this kind could accommodate about 10^3 units without difficulty. This is to be compared with the angular momentum per unit mass near the Sun (3×10^6), in the gas at 100 pc. from the galactic centre (3×10^4) and in the nucleus of *M* 31 (100). In the Fowler–Hoyle objects the maximum value would be near unity. Also the formation can extend over a rather long time and fragmentation need not be avoided. Still, the formation of such nuclei would present a formidable problem if the matter has to be condensed into stars inside the nucleus. Collisions almost certainly would disrupt the forming stars. But maybe the matter need not be condensed into very neat stars; so long as large density fluctuations and large velocities occur, shocks that become relativistic can be expected. Or it might be possible to build a nucleus from passing stars by stellar dynamical processes. In a recent paper Sweet[7] has explored some of the 'plasma type' processes in stellar systems. Many novel features appear and it is not impossible that by making use of such interactions the formation of a nucleus could be better understood. In any event, *NGC* 4151 shows that very condensed nuclei do exist, whether we understand their origin or not.

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The idea that a very close-packed set of stars might evolve through collisions has also been proposed independently by S. Ulam and by T. Gold. A paper, following much the lines discussed by Prof. Woltjer, was presented by Gold at a recent international symposium in Dallas (December 16–18, 1963).

My first comment is that, in any event in principle, Prof. Woltjer appears to be following the same path that Fowler and I took: of concentrating a very large mass into a comparatively small volume, and of then drawing on gravitational energy (through the virial theorem in this case). I accept, of course, that there are interesting and important differences in the detailed aspects of the process.

To step up the mass to $10^{10}\,M_\odot$ is still more radical than our proposal, but no worse for that. However, I do not understand the argument by which this mass is introduced. The reference to the luminosity of the nucleus of *NGC* 4151 implies a mass to light ratio typical for ordinary autonomous stars, whereas the later discussion is directed towards deriving energy from collisions between stars. The usual ratio of mass to light appears irrelevant for such a process.

I would have been interested if Prof. Woltjer had gone on to discuss the ultimate evolution of his set of close-packed stars. He avoids the two critical questions, the beginning and the end. As regards the end, I would suppose that not all the mass could evaporate, since initially the total energy is negative, and is made effectively more negative as radiation and cosmic rays are emitted. The steady destruction of individual stars should lead to the formation of one or more compact objects of large mass. If the initial mass were really as large as $10^{10}\,M_\odot$, I find it hard to see how the formation of the sort of object which Fowler and I considered, 10^6 to $10^8\,M_\odot$, could be avoided.

The sources 3*C* 48 and 3*C* 273 show light variations with a characteristic time scale of a year or more. Prof. Woltjer's calculations lead to about 10^3 star collisions a year. It is rather surprising, in view of this large rate, that the light emission should not be steady over a time scale as long as a year. Inspection of the formulæ shows that, if dE/dt is to be maintained at $\sim 10^{46}$ ergs sec^{-1}, the necessary value for sources such as 3*C* 48, 3*C* 273, there is no simple adjustment of the parameters that reduces the collision rate to ~ 1 a year.

Finally, with cosmic rays produced by a very large number of events, occurring at random positions within the system, I find it difficult to understand why observation shows emission in well-directed jets, often two jets in opposite directions. This seems easier to understand in terms of the evolution of a single compact body.

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RADIOPHYSICS

An Analytical Formula for Radio-path Determination

The shape of electron density, shown in Fig. 1, makes it possible to express a radio path by an analytical formula.

In spherical ionized layers, similar to our ionosphere, three kinds of radio propagation are to be taken into account: the penetration through the whole layer, the propagation along its inner boundary and reflexion[1].

For waves passing the whole layer, the following useful theorem has been derived[2]: before and after passing through the layer the linear parts of a ray path are merely

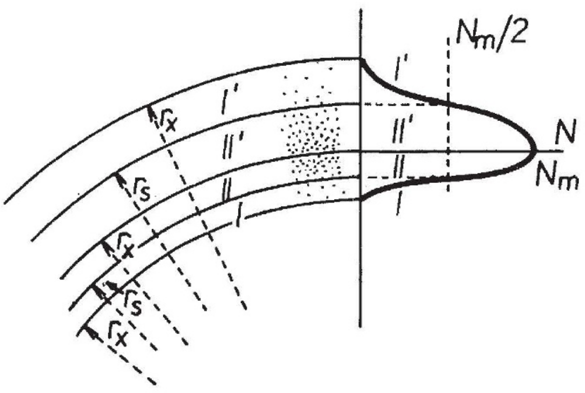

Fig. 1. $N(r)$, according to equ. (2)