Comminution: a Modified Logistic Growth Function

HOLMES's equation[1] rewritten in terms of specific surface area, S, can be derived from the differential equation:

$$\frac{dS}{dE} \propto S^{1-r} \qquad (1)$$

where E is energy input per unit volume and r is Kick's law deviation exponent[1]. Equation (1) reduces to Kick's equation[2] when $r = 0$, Bond's third theory[3] when $r = \frac{1}{2}$ and Rittinger's equation[4] when $r = 1$. These equations suggest that material of unlimited fineness can be produced by milling.

A consideration of the expenditure of energy on the material under comminution, and the utilization of energy in causing fracture, indicates that: (a) the rate of change of the energy which can lead to fracture with energy input diminishes as energy input increases; (b) as the particles decrease in size, their resistance to fracture increases; (c) S reaches an upper asymptote, S_∞, as energy input increases. These lead to a differential equation which, reduced to its simplest form, becomes:

$$\frac{dS}{dE} = k\,\frac{S(1 - S/S_\infty)}{E} \qquad (2)$$

where k is a parameter. Strictly E refers to net energy, defined as the difference between the gross energy and the energy required to maintain the mill in operation without relative movement. Equation (2) is a modified form [due to E in the right-hand side] of the Verhulst–Pearl–Reed logistic growth function[5,6]. The method involved here can be developed for differential grinding, which is analogous to competing populations.

Integrating equation (2) between non-zero limits gives:

$$E^k = A\,\frac{S}{1 - S/S_\infty} \qquad (3)$$

where A depends on the initial values of S and E, which in turn depend on the material under comminution and its previous history. However, equation (3) may be used with confidence if the initial values of S and E are small; in most instances the experimental curves of S versus E will appear to extrapolate through the origin.
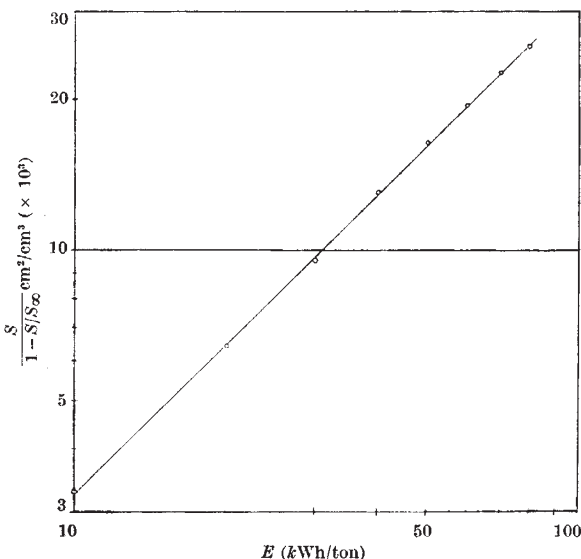
If values of S_x, S_{x+1} ... corresponding to values of E_x, E_{x+1} ... in geometrical progression:

$$\left(\frac{E_{x+1}}{E_x} = \text{constant} > 1\right)$$

are interpolated from the experimental data then:

$$\frac{1}{S_{x+1}} - \frac{1}{S_\infty} = \left(\frac{E_x}{E_{x+1}}\right)^k\left(\frac{1}{S_x} - \frac{1}{S_\infty}\right) \qquad (4)$$

which has as its solution $S_x = S_{x+1} = S_\infty$. Thus, the value of $1/S_\infty$ is obtained from the point where the line of $1/S_{x+1}$ against $1/S_x$ intercepts the line at $45°$ to the axes through the origin. This method of finding an unknown asymptote has been used in other applications[7-9].

A plot of $\dfrac{S}{1 - S/S_\infty}$ versus E on a log log grid, or a plot of S/S_∞ versus E on a logistic log grid, will give a straight line if equation (3) is applicable. If S_∞ is so large that S/S_∞ can be neglected equation (3) reduces to a power form as with Holmes.

Fig. 1 shows some data[10] treated in this manner; excellent agreement is evident. In this example $k = 1$. The value of S_∞ could depend on the material under comminution, the conditions of breakage, mill dimensions and power input.

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C. C. HARRIS

Department of Mining,
University of Leeds.

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Fig. 1. Dry grinding of ferrosilicon with 30 mm balls (series XIV) (data from Svensson and Murkes, ref. 10). $S\infty$, 36,000; k, 1; A, 3·08 × 10⁻³

OCEANOGRAPHY

Occurrence of Abnormally High Fluoride Concentrations at Depth in the Oceans

DURING the course of an investigation in these laboratories of the constancy of ionic composition of sea-water and in particular of the distribution of fluoride ion in the oceans, unusually large variations in the fluoride/chlorinity ratio have been found.

A new method[1] for the determination of fluoride in sea-waters has been applied to the analysis of more than 300 samples collected from the surface and from depth. The surface samples showed no significant geographical variation in the fluoride/chlorinity ratio (mean value about $6·7 \times 10^{-5}$), except for a few samples of high chlorinity water from the Mediterranean, in which the ratio averaged about $6·4 \times 10^{-5}$, a phenomenon probably associated with precipitation of calcium. No significant change of the ratio with depth was found in the Pacific and Indian Oceans. In the deep waters from some other localities, however, much higher fluoride/chlorinity ratios were found, frequently amounting to a 30 per cent increase.

Particularly pronounced were the fluoride/chlorinity (parts per thousand) ratios of the deep-water samples from the North and South Atlantic west of the mid-