replaced by an analogue computer and a function generator, driven by a signal proportional to the rate of change of magnetic field, the output signal controlling a variable capacitance diode in an oscillator circuit. This allows considerably greater flexibility. The low injection energy and small aperture of the Birmingham machine impose severe limits on the permissible errors of the radio-frequency at injection. As a result, the system requires a signal to noise (and drift) ratio of $10^5 : 1$, and this has been achieved. It has been found possible to accelerate deuterons readily with this installation.

Deuterium gas was supplied to the radio-frequency ion source of the synchrotron injector. With the same injection energy as for protons (460 keV.) a beam was found to enter the synchrotron vacuum system and to circulate before acceleration at a magnetic field of $\sqrt{2}$ times the value for protons. This beam was carried to full energy (estimated from the magnetic field to be 642 ± 12 MeV. for mass 2) by suitable adjustment of the radio-frequency function generator and was found to circulate at maximum field with a frequency 0·76 times that for protons. The beam was extracted from the synchrotron by the method normally used for protons[4] and was found to have an intensity of 10^8 particles per pulse over an area of 12 cm.^{2} at a distance of 16 m. from the extractor. The range of the beam in copper was measured to be 167 ± 3 gm. cm.$^{-2}$ which, allowing for 7-MeV. energy-loss in polythene and air in the beam path, corresponds to an energy of 640 ± 5 MeV., confirming that the particles accelerated were deuterons. No beam was accelerated when hydrogen gas was fed to the ion source with the radio-frequency generator set for deuterons.

The identity of the accelerated particles was also confirmed in a preliminary experiment to measure the differential cross-section for the elastic scattering of deuterons by protons. Coincidences between scattered deuterons and recoil protons from a hydrogenous target were recorded at the correct correlation angle; when the counter telescopes were moved off this angle the counting-rate dropped to 20 per cent of that previously obtained. It is hoped to continue these observations and to extend them to a study of the deuteron–deuteron interaction at the available energy.

The new radio-frequency installation was developed from a system proposed by W. Boyd and his team at the National Institute for Research in Nuclear Science, Harwell. We wish to acknowledge the valuable help we have received from them at all stages of the work. We are also indebted to D. A. Gray of the Institute for assistance in the problem of deriving signals from the rising magnetic field. Many members of the synchrotron group of this laboratory also contributed to this project, in particular K. R. Chapman, who made the necessary adjustments to the ion source, and the Counter Group, who made the range and scattering measurements.

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The Quasi-Longitudinal Approximation to the Appleton–Hartree Equation

IF an electromagnetic wave passes through a medium containing free electrons, which may make collisions with heavy particles, in the presence of a uniform magnetic field, the refractive index and absorption coefficient of the medium are given by the Appleton–Hartree equation[1]:

$$n^2 = (\mu - i\chi)^2 = 1 - \frac{X}{1 - iZ - Y_T^2/2(1 - X - iZ) \pm [Y_T^4/4(1 - X - iZ)^2 + Y_L^2]^{1/2}} \qquad (1)$$

where μ = refractive index; χ = absorption coefficient; $X = f_0^2/f^2$; $Y_T = (f_H/f)\sin\theta$; $Y_L = (f_H/f)\cos\theta$; $Z = \nu/2\pi f$; f = frequency of wave; f_0 = plasma frequency; f_H = gyrofrequency; ν = frequency of collisions between electrons and neutral molecules; θ = angle between wave normal and magnetic field.

Because of the complicated nature of the above equation the quasi-longitudinal and quasi-transverse approximations, first discussed by Booker[2], are often used.

The quasi-longitudinal approximation holds when:

$$Y_T^4/4Y_L^2 \ll (1 - X)^2 + Z^2 \qquad (2)$$

It is generally quoted as:

$$n^2 = 1 - X/(1 - iZ \pm |Y_L|) \qquad (3)$$

Equation (3) means that the wave behaves very much as if it is being propagated along the direction of the magnetic field, since, for longitudinal propagation, equation (1) reduces to:

$$n^2 = 1 - X/(1 - iZ \pm Y) \qquad (4)$$

However, equation (3) is incorrect, as can easily be seen by considering the denominator of the second term of equation (1):

$$1 - iZ - Y_T^2/2(1 - X - iZ) \pm [Y_T^4/4(1 - X - iZ)^2 + Y_L^2]^{\frac{1}{2}}$$

When the inequality (2) holds, this becomes:

$$1 - iZ - Y_T^2/2(1 - X - iZ) \pm |Y_L|$$

and $Y_T^2/2(1 - X - iZ)$ is not necessarily negligible compared with Y_L. In fact, if we assume the usual numerical condition that the inequality (2) is satisfied if the larger quantity is nine times the smaller, then $|Y_T^2/2(1 - X - iZ)|$ may be as large as $\frac{1}{3}Y_L$. The correct approximation is thus:

$$n^2 = 1 - \frac{X}{1 - iZ - Y_T^2/2(1 - X - iZ) \pm Y_L} \qquad (5)$$

and the wave does not behave as if it were propagated along the direction of the magnetic field. It will only behave thus when the term $Y_T^2/2(1 - X - iZ)$ can be neglected, that is, when:

$$Y_T^2/2Y_L \ll |(1 - X) - iZ| \qquad (6)$$

Outside these limits, but where inequality (2) still holds, equation (5) should be used. A similar correction should also be made to the expressions for the polarization.

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