A MACROSCOPIC THEORY OF INTERFERENCE AND DIFFRACTION OF LIGHT FROM FINITE SOURCES

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In the usual treatment of interference and diffraction of light, the source is assumed to be of vanishingly small dimensions (a point source), emitting strictly coherent radiation. Such a treatment corresponds essentially to an idealized wave field, created by a (classical) oscillator.

The important advances made in optics in recent years, in particular the discovery of phase contrast by Zernike[1] and of the method of reconstructed wave-fronts by Gabor[2], make it highly desirable to formulate a theory of interference and diffraction which applies to fields created by an actual (finite) source.

In papers to be published elsewhere, I propose a macroscopic theory of stationary fields produced by a finite source of natural, nearly monochromatic light. According to this theory, the intensity $I(x)$ in such a field can be expressed to a degree of accuracy sufficient for the purposes of many practical applications, in the form of a generalized Huygens's principle:

$$I(x) = \iint_{\Sigma\Sigma} \Gamma(p_1,p_2)\tau(0,x_1)\tau(x_1,x)\tau^*(0,x_2)\tau^*(x_2,x) \Lambda_1\Lambda_2^* d\Sigma_1 d\Sigma_2. \quad (1)$$

Here Γ is a factor which characterizes the source, τ is a transmission function which characterizes the optical properties of the medium and Λ is the usual inclination factor of Huygens's principle. The integration is carried out twice independently over an arbitrary surface Σ cutting across the field, the ray vectors p_i ($i = 1, 2$) at the source being related to the position vectors x_i of points on Σ by means of the canonical relations:

$$p_i = \left[\frac{\partial}{\partial \xi} S(\xi, x_i)\right]_{\xi = 0}, \quad (2)$$

S being Hamilton's point characteristic function of the medium. The factor Γ is related to the specific intensity $j(\xi)$ across the source σ (assumed to be a radiating plane element) by means of the relation

$$\Gamma(p_1,p_2) = \sqrt{\chi(p_1) \cos \theta_1}\sqrt{\chi(p_2) \cos \theta_2} \int_\sigma j(\xi) [\exp ik\{(p_1-p_2).\xi\}]d\sigma, \quad (3)$$

the terms under the square roots being unessential directional factors, which in practice can usually be replaced by a constant, $k = 2\pi/\lambda$, λ being the wave-length. We normalize Γ, setting

$$\frac{\Gamma(p_1,p_2)}{\sqrt{\Gamma(p_1,p_1)}\sqrt{\Gamma(p_2,p_2)}} = \gamma(p_1-p_2). \quad (4)$$

It is seen that in (1) the properties of the source and the transmission of the medium are completely separated; also, that (1) contains only observable quantities, the non-observable phase factors which are associated with the instantaneous emissions of light having disappeared on taking the time average. Further, it is seen that the intensity distributions produced by different sources the Γ factor of which is the same are identical. This is probably the main reason why in Gabor's remarkable method of reconstructed wave-fronts one can use a different source for the reconstruction from that employed in taking the hologram.

Equations (3) and (4) enable γ to be calculated whenever the specific intensity distribution $j(\xi)$ across the radiating surface is known. It can also be shown that, under fairly general conditions, γ and consequently $j(\xi)$ may be determined from simple experiments.

Equations (1) and (3) make it possible to obtain solutions to a variety of optical problems. Although the theory is developed for radiation in the visible range of the spectrum, it seems probable that it will also apply to electron optics and perhaps also to the optics of X-rays. In the latter it may lead to a fuller interpretation of X-ray diffraction patterns of crystals. In astronomical applications the theory may enable information to be obtained about the distribution of radiation across stellar sources from the study of interference patterns (this question is discussed in forthcoming articles by R. Fürth and E. Finlay-Freundlich, and myself, to be published in "Vistas in Astronomy" (Pergamon Press, London)), a possibility which was recognized many years ago by Michelson[3] and recently pointed out again by Hopkins[4].

In the limiting case, as $\gamma \to 1$, (1) reduces to an expression for the intensity due to an ideal point source. Thus (1) contains the mathematical formulation of practically the whole elementary diffraction theory of optical image formation as a limiting case. It can also be shown that many of the results on partial coherence obtained by earlier authors (Berek[5], Van Cittert[6], Zernike[7], Hopkins[4]) also follow from our generalized formulation of Huygens's principle.

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SEISMIC REFRACTION EXPERIMENTS IN THE INDIAN OCEAN AND IN THE MEDITERRANEAN SEA

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This communication completes the preliminary account of results of seismic refraction experiments carried out in H.M.S. *Challenger* during her world cruise of 1950–52. Five stations in the Indian Ocean continue the deep-ocean studies described in two previous publications[1], and nine stations in various parts of the Mediterranean Sea give a measure of local underwater geological structure.

Indian Ocean

Fig. 1 shows the position of seismic stations in the Indian Ocean; Table 1 gives a summary of results.