NATURE

100 YEARS AGO

The vexed question as the exact meaning of the phrase "one hour after sunset and one hour before sunrise" in the Local Government Act, 1888, referring to the lighting of bicycle lamps, was settled from a legal point of view in a Divisional Court on Thursday last. It had been held that sunset at Greenwich was meant, and the Bristol justices convicted a cyclist for riding a bicycle without a light an hour after sunset thus defined. The alleged offence was committed on August 19, 1898, at 8.15 p.m., which was less than an hour after sunset at Bristol, but more than an hour after sunset at Greenwich. An appeal was made against the decision of the Bristol magistrates; and at Thursday's Court the appeal was allowed, and the conviction quashed, their Lordships holding that the phrase in the Act referred to must not be understood to mean Greenwich time, but local time.

From *Nature* 2 February 1899.

50 YEARS AGO

The medical, social and economic problems created by a rapidly ageing population were the subject of a symposium … at the British Association meeting at Brighton on Friday, September 10, 1948. The subject was introduced by Sir Ernest Rock Carling, who reminded his audience that the great majority of the elderly are healthy and independent – they outnumber the ailing and decrepit by more than 30 to 1 – and for them the most pressing problem is how to maintain to the end of their days the standard of living of their working life. The time has come for the elderly to revolt against the convention, based on sociological, not biological, grounds, that there is a fixed retirement-age beyond which they are unfit for further work. It is a truism that chronological age is no guide to capacity in the individual; but what we want to know is, how far the increased expectation of life in the last fifty years has extended working-capacity. … Ageing begins at birth, and an athlete is 'old' at thirty-five. … Laziness, Sir Ernest maintained, is at the bottom of much facile acceptance of 'too old at forty, fifty, or sixty'. If the Civil servant must retire, why does not the clergyman? If the barrister ceases to practice, why not the judge?

From *Nature* 5 February 1949.

to establish close relations between them and cognitive processes[6,7].

Rodriguez *et al.* and Miltner *et al.* now show that local oscillatory responses can synchronize across different cortical areas — the time course and topological distribution of synchronization showed a high degree of task-related specificity. These results from humans closely resemble those obtained by intracortical recording from cats[8]. Rodriguez *et al.*[1] asked people to inspect pictures that could, on occasion, be recognized as a face (Fig. 1). They found that scrutinizing the pictures was associated with increased gamma activity over cortical regions known to be involved in visual processing. But precise phase-locking of these oscillations across cortical areas occurred only when the subjects identified a face. This state of heightened synchrony was transient. It dissolved shortly before the subjects responded by pressing a key, giving way to a second episode of phase-locked gamma activity associated with the motor response, which had a different topological distribution. The authors suggest that such dynamic changes in the phase relations between spatially distributed oscillating groups of neurons could reflect the transient formation of assemblies that are bound by synchrony and represent first perception of the stimulus and then the motor programme.

Miltner *et al.*[2] tested the hypothesis that if people learn the association between a visual and a tactile stimulus, they should form a neuronal assembly comprising cells responsive to the visual and the tactile stimuli, respectively. The authors observed a marked increase of gamma activity after the visual stimulus was presented. Notably, they also found a selective increase of gamma coherence between the visual cortex and the cortical area representing the hand that had received the tactile stimulus. This coherence must have developed as a consequence of conditioning, because it disappeared when the learnt association was lost (that is, after a sequence of visual stimuli not connected to a tactile stimulus were presented). Moreover, such coherence was not seen for signals from the area representing the non-conditioned hand. Finally, the increased coherence was confined to a narrow band around 40 Hz, supporting the idea that synchronization of oscillations in this frequency range is involved in cognitive processes.

The new results provide further evidence that synchronization might allow the selective association of distributed neurons. However, neither these nor previous studies have demonstrated this in the vertebrate brain, because we cannot yet disrupt synchronization in the relevant frequency band without affecting other response variables. But a functional role for synchronization has been demonstrated in the insect olfactory system[9]. There is also indirect evidence from psychophysical studies, which suggests that the brain binds responses together and interprets them as related if they are made synchronous by synchronizing the stimuli that evoke them[10–12]. If the brain interprets responses as related when they are made synchronous by internal mechanisms (as was the case in the people studied by Rodriguez *et al.*[1] and Miltner *et al.*[2]), gamma oscillations could well be the mechanism that binds neurons into functionally coherent assemblies.

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Mathematics

Counting up to four

Ivar Ekeland

Place marbles in a straight line. You can always arrange them so that they are not evenly spaced. But can you arrange them so that no *subset* of *k* marbles, including non-neighbours, for $k \geqslant 3$, is evenly spaced (Fig. 1, overleaf)? Again, the answer is yes: if d is the distance between the first two marbles, just put the third one at distance $2d$ from the second one, the fourth at distance 2^2d from the third, and so on, the nth being then at a distance $2^{n-2}d$ from the $(n-1)$th. But then the distance between the marbles grows extremely fast, and they are spaced more and more widely apart: the density of the alignment (that is, the proportion of occupied sites between the first and the last one) tends to zero as the number of marbles increases. What happens if we impose non-zero density? The answer is that for any integer k and non-zero density δ there will be

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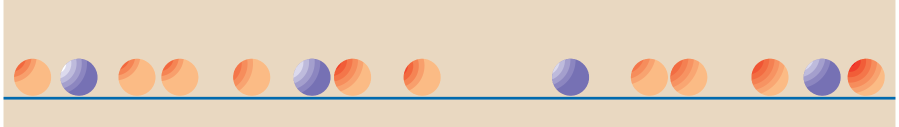

Figure 1 **A game of marbles. These marbles may look randomly distributed, but four of them (in blue) are in fact evenly spaced. A calculation of the maximum number of marbles that can be placed in a line without any four of them being evenly spaced (or reaching zero density), has just been published[5].**

an upper limit $N_k(\delta)$ to the number of marbles that can be placed in a line, without any k of them being evenly spaced, and without the density of the alignment falling below δ.

In 1936, Erdös and Turan[1] stated this result as a conjecture — that is, something they believed was true but had no proof for. Some 40 years later it was proved for the first time by Szemerédi[2]. In the meantime, Roth[3] had given a very interesting proof for the case $k = 3$. Roth's proof is remarkable in that it tells us the largest possible alignment, of given density δ, that does not contain three evenly spaced balls: the maximum number of marbles in such an alignment turns out to be $\exp\{\exp(C/\delta)\}$, where C is a constant that does not depend on δ.

Now, a double exponential is certainly a very big number, but it pales in comparison with the much larger numbers that we get from Szemerédi's proof, and from subsequent ones, such as the remarkable proof by Furstenberg[4], which relies on ergodic theory. Indeed, Roth's result was considered a notable achievement, and similar answers have long been sought for the case $k > 3$, in the hope that new estimates would lead to a better understanding of the combinations of numbers. This has just been achieved by Tim Gowers[5], who proves that any alignment with density δ that does not contain four evenly spaced marbles must have fewer than $\exp\{\exp(1/\delta^C)\}$ marbles, where C is a constant which does not depend on δ. This is a breakthrough, because Gowers's method does not seem to be limited to $k = 4$; in fact, an unpublished manuscript by Gowers extends his result to all values of k, and gives a value of $\exp\{\exp(\delta^{-\exp\{\exp(k+10)\}})\}$ as an upper bound for the number of marbles in an alignment with density δ, which does not contain k evenly spaced marbles.

Of course, mathematicians do not like being caught playing with marbles, so they phrase the problem in terms of arithmetic progressions. Start with an integer n, pick another integer r, and count in succession $n + r, n + 2r, n + 3r$, and so on, up to $n + kr$: this is an arithmetic progression of length k. For instance, all multiples of 57 lying between 10 and 500 form an arithmetic progression of length eight. In this framework, Gowers's result can be restated as follows. Let N be a natural number greater than $\exp\{\exp(1/\delta^C)\}$, and consider the set $\{0, 1, \dots N - 1\}$ of all natural numbers up to N, then every subset of size at least δN contains an arithmetic progression of length four.

Gowers's method builds upon Roth's inspiration, which was to use Fourier analysis. Roth shows that a subset A of $\{0, 1, \dots N - 1\}$ will contain many arithmetic progressions of length three unless some Fourier coefficient is appropriately large with respect to the others, in which case a simple argument yields the desired estimate. If $k > 4$, the connection between the relative sizes of the Fourier coefficients and the presence of arithmetic progressions of length k is much weaker, and this is where previous attempts at extension broke down. This is not the place to follow Gowers's argument, except to say that at some point it relies on a beautiful theorem of Freiman[6]. This relates the structure of a set of natural numbers A, provided we can estimate the size of $A + A$ (which is the set of all numbers $a + b$, where a and b belong to A), the first self-contained proof of which was provided by Ruzsa[7].

Gowers's result raises hopes that one of the most famous problems in number theory might finally be solved: does the set of prime numbers contain arithmetic progressions of any length k? In other words, given a number k, can one find an arithmetic sequence of length k consisting only of prime numbers? This question was raised by Erdös after he wrote his 1936 paper with Turan, and has remained unanswered ever since. □

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Volcanology

Fragmenting magma

Oleg E. Melnik

Explosive volcanic eruptions are among the most devastating of natural phenomena. The eruption of Santorini in the Bronze Age ended Minoan civilization; that of Vesuvius in AD 79 destroyed Pompeii; and that of Tambora, Indonesia, in 1815 killed 92,000 people. The hazards are by no means reduced these days, for as world population grows more people tend to live in areas that are potentially under threat. Hence the need to understand volcano behaviour, and whether eruptions are likely to take the explosive or gentler effusive form, and the motivation behind the work by Papale described on page 425 of this issue[1].

Qualitatively, explosive eruptions can be explained as follows. Magma with gas dissolved in it exists in a chamber in the Earth's crust, the chamber being connected with the surface by a narrow conduit. As magma ascends, pressure in it drops, the gas exsolves from the melt, and bubbles appear and grow in size. If conditions become appropriate for fragmentation of the magma, this bubbly flow changes into a gas–particle dispersion flow, and thus explosive eruption occurs.

It is this process that Papale has looked at anew. He proposes a strain-induced 'brittle fracture' mechanism for magma fragmentation, and calculates the conduit flow parameters for different types of magma using this principle. The fragmentation zone separates dense and viscous bubbly liquid below from the dilute gas–particle dispersion above. Thus the average mixture weight, conduit resistance and, therefore, the discharge rate of eruption, strongly depend on the position of the fragmentation level.

Study of explosive eruptions is difficult. They are relatively rare events, and in themselves highly dangerous, and often occur in inaccessible parts of the world. The governing parameters are known only approximately, and the characteristics of magma flow within the conduit cannot be reconstructed from indirect field measurements. Nor can the results of laboratory experiments easily be scaled-up to the natural situation because of the specific properties of magma. So we have to resort to computer simulations of eruptions to help understand their physics and forecast hazards for real volcanoes.

Simulations of conduit flow began in the late 1970s. By that time, physical models for bubbly liquid and gas–particle dispersion had become quite sophisticated but no fragmentation model for high-viscosity liquids had been put forward. In the case of vapour–water systems, which have been especially well investigated, the transition from a bubbly regime to a gas-droplet flow occurs when the bubbles coalesce into large packets. But the high viscosity of magmas prevents such coalescence and so the fragmentation mechanism must be different.

Several workers[2–4] have used an assump-

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