Difference Equations of Moment-Generating Functions for some Probability Distributions

Wishart and Hirschfeld[1] obtained the moment-generating function for the probability distribution of adjacent black-white joins when m points, black or white with probabilities p and q, are arranged at random on a line, by working out the difference equations connecting the moment-generating functions for $(m-1)$, m and $(m+1)$ points. Recently, Krishna Iyer[2] used this method to derive the moment-generating function for the distribution of black-black joins. The success of the method depends upon the fact that the r-th factorial moment, $\mu'_{[r]}$, is equal to $r!$ multiplied by the expectation for obtaining r black-black joins. This property will enable us to work out the difference equations satisfied by the moment-generating functions of a number of distributions arising from m points arranged at random on a line. This note gives *in seriatim* the difference equations for the following distributions: (i) the number of joins between points of different colours when the points can assume one of k colours with probabilities $p_1, p_2 \ldots p_k$, (ii) the number of runs of length r of a specified colour, (iii) the number of runs of length r or more of a specified colour, and (iv) the number of triplets, quadruplets, etc., of a specified colour. They are as follow:

$$\text{(i)} \quad M_m - M_{m-1} - 2a_2\theta M_{m-2} - \sum_{r=2}^{k} (r-1)a_r\theta^r M_{m-r} = 0,$$

where a_r stands for the monomial symmetric function of degree r in $p_1, p_2, \ldots p_k$, that is, $(\Sigma\, p_1p_2 \ldots p_r)$.

$$\text{(ii)} \quad M_{m+r+1} - M_{m+r} - p^rq\theta M_m + p^{r+1}q\theta M_{m-1} = 0.$$

$$\text{(iii)} \quad M_{m+r+1} - M_{m+r} - p^rq\theta M_m = 0.$$

$$\text{(iv)} \quad M_{m+1} - (1+p\theta)\, M_m + pq\theta \sum_{r=1}^{s-1} p^{r-1} M_{m-r} = 0,$$

where s is the number of points in the s-plet.

In the above expressions, p is the probability of a point taking the specified colour, and $q = 1 - p$, while θ and M_m stand respectively for $(e^t - 1)$ and the moment-generating function for m points.

By extending the methods already used by me[2], it can be shown that the cumulants of all these distributions are linear functions in m, the number of points on the line, and therefore all these distributions tend to the normal form when m tends to infinity. Details will be published shortly in the *Journal of the Indian Society of Agricultural Statistics*.

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[1] Wishart, J., and Hirschfeld, H. O., *J. Math. Soc., London*, **11**, 227 (1936).

[2] Krishna Iyer, P. V., *J. Ind. Soc. Agric. Stat.*, **1**, 171 (1948).

Sulphanilamide as a Preservative for Sweet Juices from Palm Trees

The sweet juices from palm trees (date, brab, coconut and sago) which grow abundantly in the tropics are collected for toddy (fermented juice) or for conversion into liquor. The fresh juice is found to contain about 10–14 per cent of sucrose and could be a useful source of sugar, thus replacing cane sugar and releasing fertile land for cereals.

Palm juice is fermented very quickly; to prevent inversion and fermentation while the juice is being collected, the usual practice is to add a small amount of lime to the pots. This lime does not prevent inversion and, later on, the juice is fermented. Work carried out at this Institute has shown that palm juice is slightly acid (pH about 6) when fresh, but it soon becomes acid (pH about 4), and then is fermented to produce toddy by airborne bacteria. Experiments have shown that the juice can remain fresh and unfermented for a few hours if it is kept alkaline (above pH 8); in order to preserve palm juice from inversion and fermentation, so as to convert it into either gur or sugar, a preservative is needed. It has been found that sulphanilamide, when added to palm juice in quantities 10–60 p.p.m. (40–250 mgm. to a gallon), preserves the juice in its fresh condition from five to twenty days.

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Rapid Estimation of Standard Deviation

A simple graphical method of calculating standard deviations has been described by Dr. B. Woolf[1]. It is sometimes useful, as a means of estimating short-term variations in a process or product, as distinct from long-term trends, to measure the root-mean-square of the differences between each reading and the next, a quantity which, in the absence of any trends, would be expected to have a value $\sqrt{2}\,\sigma$. A modification of Woolf's method enables this to be measured even more simply than the standard deviation.

As in Woolf's method, the various readings (in chronological order, A_1, A_2, etc.) are marked off on a line at one side of a piece of graph paper. With dividers a length A_2P_2 is then marked off equal to A_2A_1 on a line perpendicular to A_2A_1. Then $A_2P_2 = |a_1 - a_2|$. The point of the dividers is then moved from A_2 to A_3 and A_3P_3 is marked off perpendicular to A_1A_3 so that $A_3P_3 = A_3P_2 = \{(a_1 - a_2)^2 + (a_2 - a_3)^2\}^{1/2}$. By repetition of the process, moving the points of the dividers alternately, the square root of the sum of squares of differences can be found very rapidly.

Obviously, if the points have already been plotted on a control chart, there is no need to mark off the points A_1, etc.

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[1] *Nature*, **164**, 360 (1949).