reports from various medical centres. One wonders, however, why the reports are entirely from American institutions; could not one single British medical school, for example, be persuaded to contribute? On the other hand, the "Survey of Atomic Energy Developments" is well balanced and contains some information not readily available, especially regarding the work in progress in Great Britain. The section on the American effort, however, contains much administrative and organisational detail of little interest to most readers.

For those interested in the political and economic repercussions of atomic energy, there are short chapters on international control, and the financial and legal aspects. In view of the great importance of the former, one feels that insufficient space is devoted to it; in particular, an unbiased summary of the respective views of the Eastern and Western blocs on the subject would have emphasized the importance of this section of the book.

Two very useful chapters, one dealing adequately with safety precautions, and the other giving full details of the radioactive isotopes available in Britain and America, will be of special interest to those contemplating tracer and allied work; so also will the useful bibliography of books, references and journals.

One feels, in fact, that the book will more than fulfil its aim so far as those with some previous knowledge of the subject are concerned, for they will find in it much useful reference material.

D. F. BRACHER

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VECTOR AND TENSOR ANALYSIS

Vector and Tensor Analysis

By Prof. Louis Brand. Pp. xvi+439. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1947.) 33*s*. net.

THE vector analysis of Gibbs and Heaviside and the more general tensor analysis of Ricci are now recognized as standard tools in mechanics, hydrodynamics and electrodynamics. Their use not only materially simplifies and condenses the exposition, but also makes mathematical concepts more tangible and easy to grasp. Moreover, tensor analysis provides a simple automatic method for constructing invariants. Since a tensor equation has precisely the same form in all co-ordinate systems, the desirability of stating physical laws or geometrical properties in tensor form is manifest. The perfect adaptability of the tensor calculus to the theory of relativity was responsible for its original renown. It has since won a firm place in mathematical physics and engineering technology. Thus Sir Edmund Whittaker rates the tensor calculus as one of the three principal mathematical advances in the last quarter of the nineteenth century.

That this was not always the accepted view is evidenced by the following quotation from the preface to the third edition of Tait's "Quaternions": "Even Prof. Willard Gibbs must be ranked as one of the retarders of quaternion progress, in virtue of his pamphlet on *Vector Analysis*, a sort of hermaphrodite monster, compounded of the notations of Hamilton and Grassman". This robust outburst evoked a convincing refutation from Gibbs which was published in *Nature* (**43**, 511; 1891). That Gibbs has entire justification is evident, to cite a few instances, from a perusal of the "Vectorial Mechanics" of Brand (1930) and more recently of E. A. Milne (1947), and from a more general point of view E. H. Neville's "Multilinear Functions of Direction" (1921) and the extension of vectorial ideas to function space associated with the recent work of Synge, Prager and McConnell.

The present work not only comprises the standard vector analysis of Gibbs, including dyadics or tensors of the second rank, but also supplies an introduction to the algebra of *motors* which is apparently destined to play an important part in mechanics as well as in line geometry. As the notion of motor is not very widely disseminated it may be well to explain that if ε is a unit with the property $\varepsilon^2 = 0$, a *unit line vector* with Plücker co-ordinates $\mathbf{a}$, $\mathbf{a}_0$, $|\mathbf{a}| = 1$ is written as a *dual vector*

$$\mathbf{A} = \mathbf{a} + \varepsilon \mathbf{a}_0.$$

A motor is then of the form

$$\mathbf{M} = \mathbf{m} + \varepsilon \mathbf{m}_0 = (\lambda + \varepsilon \lambda') \mathbf{A},$$

where λ and λ' are scalars. The idea of a unit ε such that $\varepsilon^2 = 0$ is originally due to Levi-Civitá, who devised it in connexion with Sobrero's hypercomplex numbers $z = x + jy$, where $1 + 2j^2 + j^4 = 0$. In this notation $1 + j^2 = \varepsilon$.

In this book tensor analysis is first developed in three-dimensional space and then extended to n-dimensional space. As in the case of vectors and dyadics, the invariant tensor is distinguished from its components. This leads to a straightforward treatment of the affine connexion and of covariant differentiation, and also to a simple introduction of the curvature tensor. It is, indeed, a praiseworthy characteristic of the author's treatment that the intrinsic properties of the entities are emphasized, while the background of co-ordinates is relegated to its proper subordinate position.

The contents are as follows: vector algebra, motor algebra, vector functions of one variable, linear vector functions, differential invariants, integral transformations, hydrodynamics, geometry on a surface, tensor analysis, quaternions. In addition, there are numerous exercises at the end of each chapter.

The rich and diverse field amenable to vector and tensor methods is one of the most fascinating in applied mathematics. The author hopes that the reasoning will not only appeal to the mind but also impinge on the reader's æsthetic sense. For mathematics, which Gauss esteemed as the queen of the sciences, is also one of the great arts. Here he quotes the words of Bertrand Russell. "The true spirit of delight, the exaltation, the sense of being more than man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry. What is best in mathematics deserves not merely to be learned as a task, but also to be assimilated as a part of daily thought, and brought again and again before the mind with ever-renewed encouragement. Real life is, to most men, a long second-best, a perpetual compromise between the real and the possible; but the world of pure reason knows no compromise, no practical limitations, no barrier to the creative activity embodying in splendid edifices the passionate aspiration after the perfect from which all great work springs."

L. M. MILNE-THOMSON