The physical interpretation of the symbols and a comparison with Dingle's notation are given in the accompanying table.

TABLE OF SYMBOLS USED

Symbols used		Interpretation	
Present notation for cases (i) and (ii)	Dingle's notation for case (ii)	Case (i)	Case (ii)
a	η	Distance covered	Measure of entropy radiated
t	t	Vel. of light × time	,, ,, time
b	τ	Vel. ÷ vel. of light	,, ,, temperature
w_1	Not used	$+1$	Measure of an infinitely high temperature
w_2	$-\zeta$	-1	Constant depending on an experimental arrangement and corresponding to the absolute zero of temperature
$g(b_j)$		$(1-b_j^2)^{-1/2}$	$(1-b_j/w_2)^{-1/4}$
$h(b_j, b)$		$1-b_jb$	$1-b_j/w_2$

These properties suggest a study of the general transformations (1). They form a group provided

$$\left.\begin{aligned} g(b'_2) &= \frac{g(b_2)\,h(b_1\,,\,b_2)}{g(b_1)\,h(b_1\,,\,b_1)}, \\ h(b'_2\,,\,b') &= \frac{h(b_2\,,\,b)\,h(b_1\,,\,b_1)}{h(b_1\,,\,b)\,h(b_1\,,\,b_2)}. \end{aligned}\right\} \qquad (5)$$

Furthermore, particular values, w_n $(n = 1, 2, \ldots)$ of b such that $w'_n = w_n$ (values of b which are invariant under (1)), must satisfy

$$h(b_1\,,\,w_n) = 1 - b_1/w_n \qquad (6)$$

by (1). Hence

$$b_1 \neq w_n \quad (n = 1,2, \ldots), \qquad (7)$$

since, with $b_1 = w_n$, (6) shows that $h(w_n, w_n) = 0$, so that the transformations (1) would become meaningless in this case. If these transformations form a continuous group, it follows from (7) that there can be no value w_n of b within the range which the parameter b_1 can cover. Such values of b can thus occur only at the end-point(s) of the range of values of b_1 within which the transformations (1) are effective. In fact, with the convention $w_1 > w_2$,

in case (i) $w_1 = 1,\ w_2 = -1,\ 1 > b_1 > -1$,
in case (ii) $w_1 = \infty,\ w_2 = k,\ \infty > b_1 > k$.

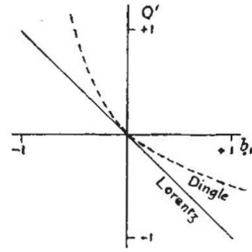
It is interesting to note that these conclusions are already inherent in the transformation (1). They represent a generalization of the property of Einstein's special relativity that, if v be an arbitrary velocity, c the velocity of light, the relations

$$v = -c, \quad -c < v < +c, \quad v = +c,$$

are each invariant under a Lorentz transformation. In Dingle's case they enable us to interpret the constant k (which occurred in the second metric) as an invariant temperature which, with the usual measures, corresponds to the absolute zero of temperature. k must be negative, since the group property of the transformation requires that the value $b_1 = 0$ be possible.

In the relativity case, b_1 is the velocity of a system of reference S' relative to another system S. Expressing this relationship symbolically, we write

$$b_1 : S \rightarrow S', \text{ and introduce } b_2' : S' \rightarrow S''. \qquad (8)$$



RELATIONS BETWEEN o' AND b_1, TAKING $w_2 = -1$ IN BOTH CASES

Hence $b_2 : S \rightarrow S''$ and, in particular,

$$\text{if } S'' \equiv S, \text{ then } b_2 = 0, \text{ and } b_2' = o' = -\frac{b_1}{h(b_1,o)} \qquad (9)$$

by (1). Hence for case (9) the relations (8) yield the reciprocal transformations

$$b_1 : S \rightarrow S', \quad -b_1/h(b_1,o) \quad : \quad S' \rightarrow S.$$

The relation between these two transformations for the two special cases is illustrated in the figure. Thus, on transforming from S' to S the last equation (1) is replaced by

$$b = \frac{b' - o'}{h(o',b')} = \frac{(b-b_1)h(b_1,o) + b_1h(b_1,b)}{h(b_1,b_1)} \qquad (10)$$

by (1), (2), (5) and (9). Using (10) to eliminate $h(b_1,b)$ from (1), our transformation is seen to be a linear transformation with matrix

$$g(b_1)\begin{pmatrix} \dfrac{1}{h(b_1,b_1)} & -\dfrac{b_1}{h(b_1,o)} \\ \dfrac{h(b_1,b_1) - h(b_1,o)}{b_1} & \end{pmatrix} \qquad (11)$$

Though we are not here concerned with the physics of the theories leading either to case (i) or to case (ii), their unification in the above way may be of interest.

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Dingle, H., *Phil. Mag.*, 35, 499 (1944); 37, 47 (1946); *Nature*, 157, 515, 556, 737 (1946).

MR. LANDSBERG's letter, in that it derives a general transformation of which the Lorentz transformation is a special case, is doubtless of importance mathematically. It seems to me to have considerable physical interest for two reasons. First, it should help to elucidate the difference between the 'one-way' character of the laws of radiation and the 'two-way' character of the laws of motion, since it shows that both sets of laws, expressed in relativistic form, are special cases of a more general relation. Secondly, the detailed correspondence obtained between the Lorentz transformation and the transformation operative in thermal relativity should help by analogy in the further development of the latter. It might help, for example, in deriving a rigorous relation between specific heat and temperature (expressed, of course, not in those terms but in terms of the thermal relativity measurements) by analogy with the mechanical relation between mass and velocity.

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