glow corresponds to an integrating term in the feed-back loop, setting an upper limit both to the frequency response and to the gain of A in accordance with Nyquist's criterion of stability[2].

If $y = F(x)$ be the equation of the curved edge relative to the *deflexion axis*, this will also (very nearly) be the equation of S, so that if S be deflected linearly with time in the X-direction, the output wave-form will be an electrical replica of the curve. More precisely, if the co-ordinates of S are (x, y), and voltages on the Y_1, Y_2, X_1, X_2 plates are p, q, r and s respectively, we have

$$p - q = \frac{y}{k_1}; \quad r - s = \frac{x}{k_2},$$

where k_1 and k_2 are normally constant, being inversely proportional to the accelerating voltage V of the tube. Hence the output voltage p will obey the law $p = q + \frac{1}{k_1} \cdot F[k_2(r - s)]$, where q, r and s (and to a limited extent, k_1 or k_2) are independent variables, and F can include rotation and displacement of the axes to which the curve was originally drawn.

Since one can also set either r or s equal to p or q, the device is capable of generating quite a variety of functions, at a speed limited only by the frequency response of the system, conditioned mainly by the type of screen employed. Special screens are available with afterglow times of the order of micro-seconds, so that a 'scan time' of the order of milli-seconds is feasible, and virtually continuous cathode-ray tube presentation of the output of a differential analyser using such a device should be possible, with obvious advantages when dealing with, for example, equations having variable coefficients. Other obvious possibilities are suggested by the ease with which functions such as $\log x$, e^x, etc., can be generated, and are at present being investigated.

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[1] Brit. Prov. Pat. Spec. No. 36849, 1946.
[2] Nyquist, *Bell System Tech. J.* (January 1932).

Latent Energy and After-glow in Flame Gases

The energy present in long-lived latent form in flame gases resides, I think, in very stable abnormal triatomic molecules formed during combustion. The amount of latent energy in flame gases resulting from combustion at constant pressure is rarely less than about 15 per cent of the heat of combustion, and, if this view is correct, the numbers of abnormal molecules must be large.

It has been suggested, by way of criticism of this, that the luminosity (after-glow) of flame gases indicates that there can only be an exceedingly small concentration of excited molecules. But this suggestion fails to account for the remarkable persistence of the after-glow in all cases where cooling of the flame gases does not take place rapidly. The numerous flame photographs taken by various observers show this clearly[1], and to the eye, of course, it persists for much longer than the photographs indicate. Indeed, after the explosion of a perfectly mixed inflammable mixture in a large vessel, it has been seen to exist for 14 sec., and there seems to be little doubt that were it possible to isolate hot flame gases away from surface, and under conditions such that cooling did not take place, it would remain for a time which could be measured in minutes. Furthermore, flame gases after being cooled to darkness by adiabatic expansion can be made luminous again by adiabatic compression[2].

It does not seem to be possible to account for the persistence of the after-glow unless it is assumed that large numbers of abnormal molecules of great stability are formed during combustion (just as was inferred from latent energy measurements), and that the *rate* at which these molecules reach the ground-state is exceedingly small. Individual molecules would not appear to possess sufficient energy for the emission of light quanta, and it seems probable that this can only happen during the infrequent collisions of sufficiently energized pairs of them. The rate at which such collisions take place, and therefore the intensity of the after-glow, depends both upon the concentration of the abnormal molecules, which varies with the amount of latent energy, and upon the flame gas temperature. This is in accord with experiment[3].

The hypothesis outlined above may also serve to explain the continuous background observed in carbon-monoxide and hydrogen flame gas spectra.

There is, I think, now no doubt that even in ordinary open flame gases some 25 per cent of the heat of combustion is in the form of latent energy and the abnormal dissociation[4] which results from it. This energy is released when the flame gases impinge upon surface.

It is hoped to publish shortly the evidence for this.

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[1] See, for example, Ellis and Wheeler, *J. Chem. Soc.*, plate v, p. 318 (Feb. 1927).
[2] *Phil. Mag.*, **9**, 399 (1930).
[3] *Nature*, **155**, 273 (1945).
[4] *Proc. Inst. Mech. Eng.*, **151**, 236 (1944).

"Turbulent Flow in Alluvium"

Sir Claude Inglis, in his letter published in *Nature* of October 19, p. 552, remarks that my formulæ for alluvial flow are based on two fundamental relationships. It would be more correct to state that two independent fundamental equations are necessary to a complete solution, and that each of these equations consists of an empirical correlation of dimensionless numbers or 'arguments', derived from the essential variables. Dr. White has produced a partial solution, which effectively ignores the Reynolds number. The following solution appears to be complete.

There appear to be seven essential variables involved, namely:

P	wetted perimeter	μ	viscosity of water
R	hydraulic mean depth	g	acceleration due to gravity
V	mean velocity		
ρ	density of water	$i = gRS$	energy gradient

These are the traditional variables. It will be observed that the density, grade and quantity of the