beans and carrots, while at the Irrigation Research Station, Griffith, work on the best methods of irrigation of both orchard and vegetable crops continued. Much of the work of the Division of Forest Products was again associated with the war effort; but the need for a more concentrated examination of timber utilization in Australia is recognized, and a series of lectures was prepared for a reconstruction training course for Service personnel proposing to enter the building trade. Wood structure, timber physics and mechanics, timber preservation and seasoning continued to receive attention, as well as the improvement of wood, particularly by gluing or impregnation with dimethylolurea.

The Food Preservation Division is to resume its investigations on the preparation and storage of chilled beef with particular attention to the loss of bloom, and work is also to be started on the rapid freezing of fruit and vegetables. Fundamental aspects of the metabolism of ripening and senescence of fruits were examined at the University of Sydney. Apparatus was designed for measuring the vapour pressure of dried foods, and heat transfer and evaporation from a wet surface under still air conditions; investigations on *Clostridium botulinium* and on the resistance of this and other bacterial spores to heat were continued. The relation between mould growth and the activity of water in fruit cake, the development of ascorbic acid in processed foods on prolonged storage and fundamental studies on the oxidation of ascorbic acid in fruit and vegetable tissue suspensions were initiated. The Division of Fisheries was able to expand its work somewhat.

The Division of Metrology continued to assist production directorates of the Ministry of Munitions and industrial organisations concerned with defence, in matters relating to inspection, design and examination of gauges, measuring equipment and precision tool-room equipment. The Division of Electrotechnology concentrated largely on confidential defence investigations, but is now considering the resumption of normal work. Its future programme is likely to be influenced by the recent formation of an Electrical Research Board. The programme of research on tropic proofing and the scope of climatic and durability tests on telecommunication materials and communications have been considerably extended. The Physics Division is also planning to extend its long-range and fundamental research; among work carried out during 1944–45 were studies of the optimum visual conditions for radar operators, the photometry of a new type of searchlight, the control and measurement of humidity in connexion with tropic proofing of service equipment, the design of an instrument for the accurate and rapid measurement of hæmoglobin for 'blood bank' work, and fundamental work on the physics of wool fibres. Aeronautical investigations have included studies of the effects of turbulence on the size, state and stability of the boundary layer formed on the surface of solid bodies moving through air, the strength and stability of plywood construction, a mathematical study of the vibration characteristics of an engine-propeller – cooling-fan combination, powder metallurgy, welding of steel tubing and fatigue failures.

In the Division of Industrial Chemistry, a Section of Chemical Physics has been established at Melbourne. The Minerals Utilization Section has perfected a method for the production of the special grade of rare-earth oxides required for polishing optical lenses and prisms. The Cement Section is investigating the deterioration of concrete through expansive reaction between aggregates and cement, and in the Biochemistry Section the development of depilatory paints less harmful to wool than the lime-sulphide paint now used, the wool-loosening action of ammonia, proteolytic enzymes and the production of 2 : 3-butylene glycol by fermentation have received attention. The separation of minerals by flotation, physico-chemical characteristics of acridine drugs and surface areas of solids have been studied by the Physical Chemistry Section. The Organic Chemistry Section was also concerned with the production of butadiene from 2 : 3-butylene glycol obtained by fermentation, and continued work on the exudation of mannitol from *Myoporum platycarpum*, synthetic resin adhesives and aniline-formaldehyde resins. In the Chemical Engineering Section pilot-plant trials were conducted of the Fremey-Lipson process rendering woollen goods resistant to felt, and of an improved process for the manufacture of rare-earth hydroxides from monazite. The general basic attack on problems associated with friction, lubrication, bearings and wear continued, including the mechanism of boundary lubrication, effect of abrasives on wear, the development of a soluble extreme-pressure cutting and drawing lubricant, the lubrication between a journal and a bearing and the behaviour of pure metals.

## THE GEOMETRY OF NUMBERS

THE meeting of the London Mathematical Society on December 19, 1946, took the form of a symposium on the geometry of numbers, arranged by Prof. H. Davenport. Prof. E. C. Titchmarsh, president of the Society, was in the chair.

In his introductory remarks, Prof. Davenport outlined the nature of the subject. It consists in interpreting geometrically questions in the theory of numbers, making use of points with integral co-ordinates, either in the plane, or, more generally, in an $n$-dimensional space. The system of all points with integral co-ordinates is called the *standard lattice*. The general homogeneous linear transformation, or affine transformation, of the standard lattice gives a *general lattice*. Thus the points of a general lattice in $n$-dimensional space are the points $(x_1, \ldots, x_n)$, where $x_1, \ldots, x_n$ are linear forms :

$$\begin{array}{l} x_1 = a_{11}u_1 + \ldots \quad + a_{1n}u_n \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x_n = a_{n1}u_1 + \ldots \quad + a_{nn}u_n \end{array}$$

in $n$ variables $u_1, \ldots, u_n$, which take all integral values. The determinant of the coefficients $a_{ij}$ is called the determinant of the lattice, and measures the density with which the lattice points are distributed in space (the density being inversely proportional to the determinant).

Many questions in the theory of numbers can be expressed in the form : Does a particular region contain a lattice point, or under what conditions is this the case ? This geometrical approach led Minkowski to many important theorems. It is also valuable in suggesting new and interesting questions, even when it does not provide any means for answering them. Two of the three topics discussed later are, in fact, treated by other methods.

Mr. C. A. Rogers spoke on his recent work[1] on a theorem of Hlawka. This theorem is a partial converse of the fundamental theorem of Minkowski, which asserts that any convex body in $n$-dimensional space, which is symmetrical about the origin of co-ordinates $O$, and has volume greater than $2^n$, contains a point other than $O$ of every lattice of determinant 1. Hlawka's theorem, in its most fundamental form, states that if any body (whether convex or not) has volume less than 1, then there exists a lattice of determinant 1 such that no lattice point, except possibly $O$, lies in the body. Mr. Rogers gave a simple proof of this theorem, restricting himself to the two-dimensional case for simplicity. We are given a bounded region in the plane, of area $A$ less than 1. Let $p$ be a large prime, and consider the lattice of points $(x, y)$ with

$$x = u/\sqrt{p}, \quad y = (su + pv)/\sqrt{p},$$

where $s$ is fixed, and $u, v$ take all integral values. This lattice has determinant 1. We give $s$ the values $0, 1, \ldots, p-1$, and suppose that each of the lattices has a point other than $O$ in the region. It can be proved that these $p$ points are all different. On the other hand, the total number of points of all the lattices in the region is easily found to be asymptotically $pA$ for large $p$. Since $A < 1$, this is impossible, and so one at least of the lattices satisfies the requirement of having no point, except possibly $O$, in the region.

Dr. H. Heilbronn spoke about some recent joint work with Prof. Davenport on a problem to which they had been led by considerations in the geometry of numbers. The problem was to find two lattice points in a given plane lattice, say $(X_1, Y_1)$ and $(X_2, Y_2)$, which generate the lattice, and for which $X_1 Y_2$ is numerically small. Two such points would be, in a certain sense, each near one of the two co-ordinate axes. Expressed analytically, the problem becomes that of investigating the minimum of a bilinear form :

$$B = (\alpha x + \beta y)(\gamma z + \delta t),$$

when $x, y, z, t$ take all integral values which satisfy the condition $xt - yz = \pm 1$. The results are as follows[2]. If $\Delta = |\alpha\delta - \beta\gamma|$, one can always satisfy $|B| \leqslant 2\Delta/(5+3\sqrt{5})$. If certain special forms are excluded (namely, those equivalent to $(x + \theta y)(z + \theta' t)$, where $\theta$ and $\theta'$ are $\frac{1}{2}(-1 \pm \sqrt{5})$ ), this can be improved to $|B| \leqslant \Delta/(4+2\sqrt{2})$. If more special forms are excluded, this can be further improved to $|B| \leqslant \Delta/(3 + 3\sqrt{2})$. But this inequality cannot be further improved by similar exclusions. There is a striking contrast with Markoff's results on the minimum of an indefinite quadratic form $(\alpha x + \beta y)(\gamma x + \delta y)$. There Markoff established the existence of an infinite sequence of possible minima. In the present problem, the first two possible minima of the bilinear form correspond to those of a quadratic form, but the third does not.

Prof. Davenport gave the third talk, which was on another joint paper[3] with Dr. Heilbronn. Let $Q(u_1, \ldots, u_r)$ be an indefinite quadratic form in $r$ variables. It has been conjectured for many years that if $r \geqslant 5$, such a form takes arbitrarily small values for an infinity of integral values of the variables. If the form has rational coefficients, this is true, since it then represents zero infinitely often. The general conjecture has an interpretation in the geometry of numbers. It is that the region of $r$ dimensional space defined by

$$|x_1^2 - x_2^2 \pm x_3^2 \pm \ldots \pm x_r^2| < 1,$$

where $r \geqslant 5$, contains a point other than $O$ of *every* lattice. In the language of Mahler, this region is of 'infinite type'.

A special case of the conjecture was proved by Chowla in 1934, namely, when $Q = \lambda_1 u_1^2 + \ldots + \lambda_r u_r^2$ and when $r \geqslant 9$. In the present work the result is proved for this particular form when $r \geqslant 5$. The proof uses a modification of the powerful methods created by Hardy and Littlewood for the treatment of problems in additive number-theory.

Prof. L. J. Mordell expressed appreciation of the talks, and said that the work of Mr. Rogers was particularly interesting in view of the complication of Hlawka's original proof, and of the deep ideas associated with the theorem by Siegel, in his recent proof in the *Annals of Mathematics*. Mr. Rogers had shown the value of a new and elementary approach to an apparently difficult theorem.

[1] *Ann. Math.*, in the press.
[2] *Quart. J. Math.*, in the press.
[3] *J. London Math. Soc.*, in the press.

# COMPLETE DOCUMENTATION*

By Dr. S. C. BRADFORD

THIS Conference is important historically, because it marks the beginning of a second stage in the progress of documentation. In the past, our main endeavour has been directed to bring about the collaboration of documentary agencies in the single task of making accessible the scattered records of thought and achievement, especially in science and technology, until to-day more decimally classified references are produced than different papers abstracted. But, as the result of statistical investigations, made in the main by the British National Section, we have come to see that this alone will not suffice to produce a tolerably complete index to the records of human endeavour. Half the records are overlooked by the existing abstracting and indexing agencies. We need a means of covering all this literature. The means has been discovered. Its promotion has commenced, and this begins a second stage in our activities.

Statistical analyses have been made of the whole intake of the Science Library, London, not merely of three per cent samples. The results are for that reason the more reliable, although there is still room for more research, for which financial aid would be desirable.

The investigations were divided into four parts : (i) It was established that about $\frac{3}{4}$ million useful scientific and technical papers are recorded every year in an aggregate of some 15,000 useful scientific periodicals, while 300 abstracting and indexing periodicals publish also $\frac{3}{4}$ million abstracts, or references. But these abstracts, or references, relate to only $\frac{1}{4}$ million different papers. So that more than half the useful discoveries and inventions are recorded, only to be buried on the library shelves.

(ii) This general analysis was confirmed by the examination of the literature of certain definite subjects. The details of the examination of electrical engineering literature show that only 11,500 different papers on this subject, out of an annual output of 24,000, are covered by a yearly total of 45,000

* Synopsis of a paper read at the sixteenth International Conference of the International Federation for Documentation at Paris, November 4–9, 1946.