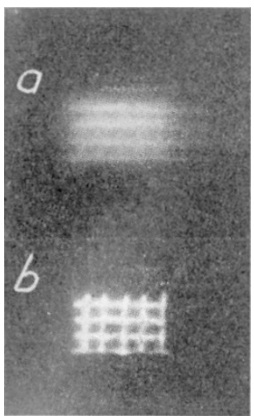
In order to obtain the desired field form, where the field strength in the ring-shaped pole gap varies as $1/\sqrt{r}$ in as large an area as possible, it was found convenient to use semi-empirical methods. If no regard is paid to the inward and outward stray fields the pole profiles should be given a parabolic form. By measuring the inhomogeneity of the field before and after grinding the pole shoes a certain amount, it is possible to compute how the pole profiles should be ground to reduce the influence of the stray fields inside the pole gap. After three grindings, a distance of 26 mm. in the 44 mm. wide pole gap was obtained, which possessed accurately the required form of field.



The double focusing property of the spectrograph was studied in the following way. A wire net was activated with thorium B and placed in the spectrograph. The current through the magnet was adjusted for focusing of the F-line ($H\rho = 1383{\cdot}8$) which is dominant in this β-spectrum. Before the slit of the Geiger–Müller tube of the spectrograph, which was situated 256° from the wire net, a photographic plate was placed. The result of an exposure is shown in the reproduction where (a) is the image of the wire net in the spectrograph and (b) is a contact photograph of the same net for comparison. Besides the vertical meshes, the horizontal meshes are quite well reproduced. These would, of course, not have appeared if a semicircular method had been employed.

In view of the great possibilities of this focusing system, a larger apparatus has been planned. The radius of curvature will be 50 cm. and the employable solid angle will be more than 0·1. The weight of the iron will not exceed 2 tons.

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[1] Svartholm, N., and Siegbahn, K., *Ark. f. Mat. Astr. Fysik*, in the press.

An Attempt to Formulate a Divergence-free Quantum Mechanics of Fields

I HAVE attempted to introduce in a certain way a constant a of dimension length in order to formulate a divergence-free theory of interaction. The functions of field ψ,ψ^* are treated not as Fourier series:

$$\psi = \sum_k \psi_k, \text{ where } \psi_k = A_k e^{i\vec{k}\vec{x}} + B_k e^{-i\vec{k}\vec{x}}, \quad (1)$$

(we have taken as an example the scalar meson field) but as matrices: $\psi = \|\psi_{kl}\|$, where

$$\psi_{kl} = \frac{1}{\sqrt{2}}\left\{(A_k{}^l e^{i\vec{k}\vec{x}} + B_k{}^l e^{-i\vec{k}\vec{x}}) + i\,(A_l{}^k e^{i\vec{l}\vec{x}} + B_l{}^k e^{-i\vec{l}\vec{x}})\right\}. \quad (2)$$

The unsymmetrical part in the first (or second) brackets could as well be used as ψ_{kl}; we have taken both (with the factor $1/\sqrt{2}$) on account of symmetry.

Densities of energy, charge, etc., which are always quadratic in field variables ψ,ψ^* are defined as traces of corresponding matrices, for example:

$$\psi^*\psi = Tr \,\|\, \psi^*\psi \,\| = \sum_{kl} \psi^*{}_{kl}\,\psi_{lk}. \quad (3)$$

In analogy with the usual commutation relation:

$$[\,\psi^*(x'_1t'),\ \psi(x_1t)\,] = \frac{h}{i} D(x' - x, t' - t), \quad (4)$$

we assume:

$$Tr[\,\psi^*(x'_1t'),\ \psi(x_1t)] = \frac{h}{i} D(x' - x, t' - t). \quad (5)$$

While in the usual theory the relation (4) fully determined the Fourier coefficients A_k, B_k, $A_k{}^*$, $B_k{}^*$, we have in our case a certain freedom in choosing the coefficients $A_k{}^l$, $B_k{}^l$ . . . (5) may be satisfied by putting:

$$A_k{}^l = A_k e^{-a^2|(\vec{k} - \vec{l})^2|}, \quad (6)$$

where A_k means the well-known matrix of the usual theory, $\vec{k},\vec{l}$ are four-vectors and a is a constant. It is possible that a is the well-known 'universal length'. We see at once that the diagonal elements ψ_k are identical with the usual Fourier elements ψ_k and that the modified theory becomes identical with the usual one for $a \to 0$. Instead of $e^{-a^2|(\vec{k}-\vec{l})|}$ we might have put another function of argument $(\vec{k}-\vec{l})^2$, which is unity for $\vec{k} = \vec{l}$ and tends to zero for high values of $|(\vec{k} - \vec{l})^2|$.

New expressions for densities of energy, charge, etc., result from this procedure. While in the usual theory it was:

$$\psi^*\psi = \sum_{kl} \psi_k{}^*\psi_l, \quad (7)$$

in the modified theory it is

$$\psi^*\psi = Tr\,\psi^*\psi = \sum_{kl} e^{-a^2|(\vec{k} - \vec{l})^2|}\,\psi_k{}^*\psi_l. \quad (8)$$

This modification does not influence the integral values of the respective observables in the case of a field *in vacuo*: charge remains the integral multiple of the elementary charge, energy the integral multiple of $ch\sqrt{u^2 + k^2}$, etc. But it is otherwise with the expression for interaction energy between two fields: our modification changes this expression by causing a relativistically invariant damping of interaction. In case of interaction between electronic and electromagnetic fields we keep the usual formula

$$\mathbf{H}_{(i)} = -\frac{1}{c}\sum_\mu s_\mu \Phi_\mu, \quad (9)$$

but define, of course, s_μ and φ_μ as traces of corresponding matrices. Thus we get for the matrix element H_{kl} describing the probability for transition from state l to k:

$$H_{kl} = e^{-2a|(\vec{k} - \vec{l})^2|}\, H'_{kl},$$

where H'_{kl} means the matrix element of the usual theory. Higher degrees of perturbation calculus yield finite results; in particular, the self-energy of an electron in an electromagnetic field becomes finite. If we assume $a = 1/137\,\mu$ the electromagnetic self-energy is about 20 per cent of the total energy of electron.

In case of fields without negative energy states $\vec{k} - \vec{l}$ is a space four-vector. Then our modification is equivalent to the introduction of an operator $e^{a^2\Box}$ to modify the definitions of densities:

$$s_\mu = e^{a^2\Box}\, s'_\mu, \quad (10)$$

where s'_μ is the density of current in the usual theory. s_μ is a four-vector satisfying the equation of continuity, since $e^{a^2\Box}$ is a scalar operator which commutes with Div. Of course, the operator $e^{a^2\Box}$ is not uniqu . Prof. Rubinowicz directed my attention to another, namely, $e^{-a^4\Box^2}$, which yields the damping factor $e^{-a^4(\vec{k}-\vec{l})^4}$ and seems better than the former. Both are in some way connected with the Gaussian error function describing the probability of finding the value k in the case when the mean value has been l (or vice versa) and the uncertainty $\Delta k = 1/a$.

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Hole Theory of the Liquid State

A QUASI-CHEMICAL theory of the liquid state can be built up by incorporating the cluster idea of the Band–Frenkel theory of condensing systems into the Eyring theory; thus the free volume of the liquid molecules will become an adjustable parameter and a function of temperature. A similar theory had already been advanced by Fürth[1], who considered the liquid as a continuum permeated by holes, and attributed the properties of the liquid as due to their presence. Recently, Auluck and Kothari[2] have shown that the energy-levels are discrete on solving the appropriate Schrödinger equation for the hole by BWK method. However, we believe that a classical approximation may be still valid if the temperature of the liquid is sufficiently high above the melting point.

The 'negative' clusters so formed possess physical significance. Their real existence should be interpreted in the dynamical sense of the word; that is, a balance between association and dissociation of them will be reached when the liquid is in the equilibrium state. At any temperature, there is a set of numbers defining the statistical-mean-numbers of clusters of various size, and at the critical point the assembly may be pictured as a liquid permeated by 'negative' clusters, or as a vapour composed of 'positive' clusters alike in equal numbers and sizes. This assumption appears quite permissible, as evidenced by the diffused pattern given by X-ray diffraction in a liquid. These