

The combination of these two new ideas produced the now famous Fejér-Lebesgue theorem, which shows, in modern language, that summability $C,1$ is a 'Fourier-effective' process; that is to say, it sums the Fourier series to the 'correct' sum almost everywhere. This, it is worthy of note, is in marked contrast to classical convergence; in fact, Kolmogoroff has recently demonstrated the existence of Fourier series which are *nowhere* convergent.

F. Riesz and E. Fischer, working independently, were finally able to formulate a precise converse of the old formal result known as Parseval's theorem (1799). The existence theorem bearing their name is to the effect that, if $\sum_{n=1}^{\infty} (a_n^2 + b_n^2) < \infty$, there exists a

measurable function, with an integrable square modulus (that is, a function of the Lebesgue class L^2) with these numbers as Fourier constants, and furthermore that the partial sums of the Fourier series converge *in mean* to this function. With reference to this it should be observed that the Lebesgue integral plays an indispensable part in the proof; in fact, the theorem is *false* for any definition of the integral narrower than that of Lebesgue.

With these fundamental ideas in view, the reader will find in this tract an admirably lucid and precise account of most of the major developments which have taken place during the last forty years. The authors have used the Lebesgue definition of the integral throughout, at the same time indicating those theorems which remain true, *mutatis mutandis*, if Riemann or Cauchy integrals be used. To say that a given trigonometrical series is a Fourier series is to say that a certain set of integral equations has a solution, and the meaning of such a statement plainly depends on the type of integral used. A definition wider than that of Lebesgue, for example, would in general increase the class of functions possessing Fourier series, just as a narrower one would decrease it.

After a preliminary chapter consisting principally of introductory concepts, there follows a discussion on orthogonal systems of functions of L^2 . Any theorem proved for a series of such functions is true *a fortiori* for an *ordinary* Fourier series in virtue of the orthogonality of the sequence $\{e^{inx}\}$, and this affords an interesting (and indeed valuable) logical approach to the theory. In most cases such theorems on Fourier series are admittedly deducible by independent (though not necessarily more elementary) methods and, where possible, the authors have provided alternative proofs.

The standard tests for convergence, those of Dini, Jordan, de la Vallée Poussin and Lebesgue, and their relation one to another, are then dealt with, and their analogues for the conjugate series (not necessarily itself a Fourier series) are also provided.

Following Toeplitz, the authors then investigate the application of generalized summation processes to trigonometrical series; in particular the Cesàro and Abel methods, both possessing positive kernels, are shown to be Fourier-effective.

Finally, a chapter dealing with 'uniqueness' theorems is supplied, culminating with the result of de la Vallée Poussin that if a trigonometrical series converges, except possibly in an enumerable set, to a finite integrable function, then it is necessarily the Fourier series of that function.

The entire tract is a model of clarity and precision, the authors having spared no pains to ensure that the reader shall never be at a loss to follow them, even

through their most intricate arguments. It is true that there are one or two paragraphs (for example, in the proof of Gergen's modified form of Lebesgue's convergence test, and again in the construction of a Fourier series which diverges almost everywhere) where the logic is convincing, but where a little explanation of the reasons underlying the mode of building up the argument would aid comprehension; but for this omission we must doubtless blame the severe compression, without which it would have been impossible to display such a wealth of valuable material within the short space of a hundred pages.

This tract cannot fail to be of inestimable value, particularly as a 'curtain raiser' to Zygmund's standard treatise.
J. H. PEARCE.

AUSTRALIAN ORCHIDS

The Orchids of New South Wales

By the Rev. H. M. R. Rupp. Pp. xv+152. (Sydney: National Herbarium, Botanic Gardens, 1943.) 9s. net.

WHEN the first handbook of the New South Wales flora was published in 1893 it contained descriptions of one hundred and seventy-three orchids, whereas in the work under review the Rev. H. M. R. Rupp provides descriptions of no less than two hundred and forty-eight. The large majority of these orchids are terrestrial species, only fifty-two being epiphytes, and they include a number of interesting genera, among which may be cited *Prasmophyllum* and *Cryptostylis*. Of the former, rather more than half the eighty known species are dealt with here, while of the latter twenty species are known and three occur in New South Wales. In all the members of these two genera the inferior ovary, instead of exhibiting the half-twist through 180° , as in most orchids, undergoes a complete twist during development so that the flower has a normal orientation but is reversed compared with most orchid flowers. One naturally thinks of the analogy with the leaves of *Alstroemeria*, where most species exhibit a twist of the base that brings about complete reversal, while in a few an edge-on position of the leaf is assumed as the leaf-base only undergoes a half-turn. The changes, both anatomical and morphological, which accompany such complete reversal, may be profound, and the fact that these ensue, rather than, what would appear simpler, namely, complete suppression of twisting, suggests a sort of momentum in evolution, since further genetical changes in the same direction appear to be more readily achieved than such as would be accompanied by a reversion to the ancestral condition.

The text of this work furnishes keys to the genera and species, and descriptions of the latter, accompanied by an account of their distributions. The twenty-three full-page figures illustrate the habit and floral structure of some of the more important types. The first descriptions of more than thirty of the species of this region we owe to the author, which gives some indication of the extent to which this aspect of systematic botany in New South Wales is indebted to his studies.

The Rev. H. M. R. Rupp belongs to that honoured band of gifted amateurs who have devoted their leisure to taxonomic studies, and the present volume is a valuable contribution to the new Flora of New South Wales to which the author looks forward.

E. J. SALISBURY.