Ancient Hindu Algebra

History of Hindu Mathematics: a Source Book. Part 2: Algebra. By Bibhutibhusan Datta and Avadhesh Narayan Singh. Pp. xvi + 314. (Lahore: Motilal Banarsi Das, 1938.) 7.8 rupees; 13*s*.

THIS is the second volume of the three in which the authors are writing the history of Hindu mathematics. The first contained Part 1, dealing with numeral notation and arithmetic, which was issued in 1935, and noticed in NATURE of January 18, 1936, p. 88; the present volume (Part 2) contains the history of algebra.

Historians of mathematics have hitherto been handicapped in describing the course of development of Indian mathematics because they have generally been dependent upon a few special studies such as H. T. Colebrooke's "Algebra with Arithmetic and Mensuration from the Sanscrit of Brahmagupta and Bhaskara" (1817) and new editions of that work, "Notes on Indian Mathematics" of various dates by G. R. Kaye, a study by the same scholar of the Bakhshālī manuscript, and a few original texts and German or English translations, mostly brought out in India and so not generally accessible. But the published material is small compared with the extent of the sources. The editors' bibliography in Part 1 contains some thirty-eight items setting out the various books and MSS. to which they have had recourse, and from which they have translated in full a vast number of passages. They estimate that about half the content of this second volume is here presented for the first time, so that the future historian will have at his disposal copious material as never before.

The volumes are definitely source-books, and in consequence by no means easy reading. One characteristic of the Indian mathematicians is the careful formulation, *in extenso*, of *rules* for the solution of every type of algebraical problem (whereas the remains of the ancient Egyptian and Babylonian mathematics contain a multitude of examples, but practically no expressed rules). The rules have all to be expressed in words because the Indian had practically no symbols; the unknown was "yāvat-tāvat" or "avyakta", and was denoted sometimes by a blank with the number representing the coefficient following it, sometimes by abbreviations for the names of various colours, sometimes by letters of the alphabet; signs for operations were limited to + (after the symbol to which it relates), or a superposed dot or small circle, for minus, the others being abbreviations for words; powers were 'square', 'cube', 'square-square', 'square-cube', 'cube-cube', so that the Indian symbolism had not even in Bhāskara II's time (twelfth century A.D.) advanced beyond the stage represented by Diophantus (third century). It is not surprising, therefore, that the statement of a rule for solving an equation or a set of equations sometimes runs to many lines, equal to a third or a half of the printed page, in one of these volumes.

The subjects include (besides technical terms, laws of signs, fundamental operations, etc.) linear equations in one, two, or more unknowns, quadratic equations, single and simultaneous, a few equations of higher degrees, indeterminate equations of the first degree, simultaneous equations of the first degree, the solution in integers of the equations $Nx^2 + 1 = y^2$ ("square-nature") and $Nx^2 \pm c = y^2$, general indeterminate equations of the second degree, rational right-angled triangles, rational quadrilaterals, linear functions made squares or cubes, double equations of the first and second degrees, and finally the solution of the equation $axy = bx + cy + d$.

The genius of the race for the theory of numbers brought the Hindus by A.D. 1150 to their supreme achievement, the general solution in integers of the indeterminate equation of the first degree in two unknowns and of the famous (miscalled) "Pellian" equation. The first was solved by the method called the 'pulveriser' ('kuṭṭaka'), and the stages through which the solution passed are represented by Aryabhaṭa I (born A.D. 476), Bhāskara I (522), Brahmagupta (628), Mahāvīra (850), Aryabhaṭa II (950), Srīpati (1059) and Bhāskara II (1150). The editors give the rules stated by each of these writers. The importance attached by the ancient Hindu mathematicians to the method of the 'pulveriser' is attested by the fact that the whole science of algebra was once named after it ('kuṭṭaka-ganita", the science of 'kuṭṭaka'). Still more remarkable is the solution in integers of $Nx^2 + 1 = y^2$ (the 'square-nature'). The account begins with Brahmagupta's Lemmas, rediscovered by Euler in 1764 and by Lagrange in 1768, and finishes with the 'cyclic' method described by Bhāskara II (1150). Fermat, Brouncker, Wallis and Euler could do no more than rediscover what the Hindus had given, and only Lagrange supplied the final proof that the 'cyclic' method will in every case produce the desired result whenever N is a number which is not a square.

The book has evidently involved untold research and labour, and cannot be too strongly commended.