kind or other formed in the flame front[5]. For many purposes the term 'combustion level' is a more convenient one, by which is meant the ratio of the energy which can be accounted for in flame gases on the assumption that they are normal gases to the heat of combustion of the inflammable gaseous mixture.

Measurements show that the combustion level in ordinary flame gases (which are assumed to be similar to those left behind the initial slow movement of flame during its passage through inflammable mixtures at atmospheric pressure) is of the order of 72 per cent in carbon monoxide – oxygen – argon mixtures, 80 per cent in carbon monoxide – air mixtures, 83 per cent in methane – air mixtures and 85 per cent in acetylene – air mixtures[6]. The combustion level in flame gases left behind a 'detonating' flame front would approach 100 per cent owing to the high pressure in the flame front.

The combustion level in any given mixture may apparently have any value between some lower limit and a higher limit approaching 100 per cent. The not very intense luminosity of many oxy-gas flames suggests much lower combustion levels than those measured in air-gas flames.

W. T. DAVID.

Engineering Department,
University,
Leeds.
Dec. 9.

[1] *Phil. Mag.*, **18**, 307 (1934); **22**, 513 (footnote) (1936).
[2] *Phil. Mag.*, **21**, 280 (1936); **22**, 513 (1936); and in press.
[3] NATURE, **130**, 930 (1932); and *Phil. Mag.* (in press).
[4] *Phil. Mag.*, **9**, 390 (1930).
[5] NATURE, **138**, 930 (1936); also *Phil. Mag.* (in press).
[6] *Proc. South Wales Inst. Eng.*, 375 (1936).

Wave Mechanics of Couples (Neutron – Neutrino)

FERMI considers the β-decay as the transition of a 'heavy' particle from the neutral state (neutron) into a charged state (proton) with simultaneous emission of two light particles (electron, neutrino). I consider correspondingly the β-decay as a special case of processes in which a *couple of particles* (neutron — neutrino) are involved. The principal feature of these processes is the fact that the rest-mass of the single particle is not conserved, but only that of a couple as a whole. The accepted methods of quantum mechanics do not apply to this case; some new principle is required.

I reject the idea of introducing rest-mass as a *new* observable, since there is no variable known on which the corresponding operator should be applied. I shall show that a straightforward generalization of Dirac's method leads to a consistent theory of a couple of particles.

Let the co-ordinates of the two particles (in four dimensions; $x_4 = ict$) be x_k, X_k ($k = 1, 2, 3, 4$), the corresponding momenta $p_k = 1/i\ \delta/\delta x_k$, $P_k = 1/i\ \delta/\delta X_k$. Let $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ be four Dirac matrices, $\gamma_k\gamma_l + \gamma_l\gamma_k = 0$ ($k \neq l$; $k, l = 1, 2, 3, 4$), chosen as real ($\gamma_k^+ = \gamma_k$) and normalized ($\gamma_k^2 = 1$).

Then the quantities $\gamma_{kl} = i\gamma_k\gamma_l$, $\Gamma = -\gamma_1\gamma_2\gamma_3\gamma_4$, $\Gamma_k = i\Gamma\gamma_k$ are also real and normalized. We construct the most general operator Ω which is (1) a real number in the hyper-complex algebra of the γ's, (2) linear in the p_k, (3) linear in the P_k, and for which (4) $\Psi^+\Omega\Psi$ is a relativistic invariant ($\Psi(x,X)$ is a wave function with four components forming a column, Ψ^+ its adjoint, forming a row); the form of Ω is uniquely determined, as

$$\Omega = A + B\,\Sigma_k\gamma_k p_k + C\,\Sigma'_{kl}\gamma_{kl}(p_kP_l - p_lP_k) + D\,\Sigma_k\Gamma_kP_k + E\Gamma,$$

where A, B, C, D, E are real constants.

In analogy with Dirac's theory, we subject Ω further to the

Postulate: Ω^2 is free from spin quantities.

Since Ω^2 must be invariant, it must have the form

$$\Omega^2 = A^2 + B^2\Sigma_k p_k^2 + C^2\Sigma'_{kl}(p_kP_l - p_lP_k)^2 + D^2\Sigma_kP_k^2 + E^2.$$

The analysis of this postulate leads to the

Theorem: The necessary and sufficient condition for Ω^2 being free from spin quantities are the relations

$$A = 0,\quad BD - CE = 0.$$

They reduce the five constants to three. As the dimensions of Ω do not matter, we can choose one constant as unity, say, $E = 1$. For the only two arbitrary constants left we introduce the notations $B = \lambda_0$, $D = \Lambda_0$ (Compton wave-lengths); then $C = \lambda_0\Lambda_0$, and

$$\Omega = \Gamma + \Lambda_0\Sigma_k\Gamma_kP_k + \lambda_0\Sigma_k\gamma_kp_k + \lambda_0\Lambda_0\Sigma'_{kl}\gamma_{kl}(p_kP_l - p_lP_k).$$

Here p_k and P_k have to be replaced by $p_k + \alpha\Phi_k$ and $P_k + \beta\Phi_k$, where the electromagnetic potentials Φ_k are symmetric functions of $x_k - X_k$ and α, β operators which transform any function of x_k, X_k into its symmetric resp. antisymmetric part. Then the wave equation can be derived from the variation principle

$$\delta\int\Psi^+\Omega\Psi\,dx\,dX = 0.$$

For treating an assembly of particles one has to consider Ψ, Ψ^+ as observables (non-commuting quantities), and to apply the method of second quantization.

I have proved that this theory represents the electromagnetic forces and the nuclear 'exchange' forces by the same formalism.

MAX BORN.

University,
Edinburgh.
Nov. 26.

Interaction by Resonance of Radio Waves

DR. MARTYN and I have already given a quantitative theory of the phenomenon known as the interaction of radio waves[1]. This theory has been found to be in good agreement with observation, and has in particular led to conclusions about the acoustic distortion of the impressed modulation which the subsequent observations of van der Pol and van der Mark adequately confirmed. There is, however, in this theory an inaccuracy of which I became aware about a year ago while engaged in a related investigation, and which at first sight appears of no great importance.

On revising the theory and establishing it on a more rigorous foundation, however, an unexpected and interesting consequence emerged, namely: that in regard to the amount of modulation which a wave W can impress on another wave W', there may occur