The Theory of Dimensions

The Theory of Dimensions and its Application for Engineers
By Dr. F. W. Lanchester. Pp. xxiv+314. (London: Crosby Lockwood and Son, Ltd., 1936.) 12*s*. 6*d*. net.

"THE first business of an author," says Dr. Lanchester in the preface, "is to state clearly the object of his work. . . . This book is written with the primary object of helping the young engineer to acquire a sound knowledge of dimensional theory, a subject which he can no longer afford to ignore." Although the subject forms a part of the curriculum of the engineering courses in all universities, the author maintains that subversive doctrines are being taught, and one of the avowed objects of the book is "to extirpate heresy of this kind", a job which he sets about with evident zest.

Of the 314 pages, about a hundred are devoted to tables of physical constants of the atmosphere, water, steel, fuels, etc., weights and measures, and although they contain much useful information, many of these chapters make little or no mention of dimensions. The rest of the book may be divided into two parts, mechanical and electrical. In the former, the author is evidently on his native heath; he hits out with no uncertain aim and speaks with an authoritative air. In the latter he is admittedly an explorer, feeling his way, asking directions from others, and quoting their opinions. As he says, "No attempt is made in the present work to decide between one system and another; the author has endeavoured to keep an open mind—only he declines to subscribe to any system that does not conform to his axiom," the axiom being that any one physical entity cannot have assigned to it more than one dimensional expression. In the preface he makes the somewhat enigmatic statement that "the fundamental dimensions recognised by physicists are (1) length, (2) mass, (3) time. There are really no others." It is not quite clear whether this is intended to mean that if physicists did recognize any other they would be sinning against a Lanchester axiom, but this doubt is dispelled when, with reference to Everett's statement that the choice of fundamentals is a matter of convenience, he says that it may be so, but, if true, is of little more than academic interest.

After showing that the entity and its dimensions are not one and the same, and that even the concept of velocity is not necessarily associated with length and time, but only becomes so by our definitions and methods of measurement, the author launches into an attack on the 'slug', a unit of mass introduced about fifty years ago by Prof. Perry and apparently still used by some misguided folk. A five-page appendix on the same subject is a sermon on the text, "the evil that men do lives after them".

The chapters dealing with the application of dimensional theory to a large variety of mechanical problems are excellent. In discussing thermal entities, the author regards entropy as a dimensionless numeric, whereas Kaye and Laby give it dimensions; temperature is regarded as having dimensions L^2/T^2, and the heat imparted to a mass to raise its temperature becomes ML^2/T^2, the dimensions of energy. This matter is also discussed in an appendix.

Only forty pages of the book, excluding appendixes, are devoted to electric and magnetic dimensions, but the whole of Rücker's paper on "The Suppressed Dimensions of Physical Quantities" is reprinted as an appendix, as is also FitzGerald's paper on "The Dimensions of Electromagnetic Units", and a paper by Sir J. B. Henderson on "Fundamental Units in Electrical Science".

It is on p. 104 that the author leaves his native element and enters the electromagnetic world. In his opening sentence he says, "Electrical entities depend upon the three fundamentals L, M, and T, as do other derived physical entities discussed in the preceding chapters". It would have been more explanatory of the history of the subject if he had said that they may be made to appear to depend upon L, M, and T, by making certain arbitrary assumptions about which scientific workers have been squabbling for the last fifty years. The same is true of the subsequent statement that "it is agreed by all authorities that electrostatic units and electromagnetic units have different dimensional values although the entities represented may be the same". Is it not that they are made to appear to have different dimensional values by the adoption of different arbitrary assumptions? The electrostatic units of inductance or capacitance may differ largely in magnitude from the corresponding electromagnetic units, but they cannot really have different dimensional values.

Although the author expresses strong disapproval of the unit pole, he has not been bold enough to free himself entirely from it. Much of the confusion and uncertainty which pervade the electric and magnetic units is due to the fact that the classical system is based in part upon an ill-defined unreality, namely, the pole strength of a permanent magnet. If we assume that ferro-, para-, and dia-magnetism are all due to the orbital movements of electrons superposing their effects upon the externally applied magnetomotive force, we are justified in regarding all magnetic phenomena as electromagnetic, and as being manifested in 'non-magnetic' space. In iron, the value of H is enormously increased by the molecular currents, but the relation between B and H is really the same as in air. It is usual, however, among engineers to retain the symbol H for that small component of the magnetizing force which can be ascribed to the externally applied ampere-turns, and to ask no question about the molecular mechanism whereby it produces in iron a magnetic induction B so much greater than in air. In discussing the fundamental relations between electric and magnetic concepts it is very desirable to exclude such complex phenomena as the magnetization of iron and steel and, above all, such a fictitious complexity as a permanent magnetic point-pole of unit strength.

If one must have a magnet of unit pole strength, let it at least be an electromagnet—a coreless solenoid, the current in which can be adjusted to the correct value: a much more convenient procedure than stroking a knitting needle the requisite amount with a bar magnet, even in imagination. Moreover, one is not then likely to pretend that, having measured the force on the unit pole when placed at a given point in the magnetic field, one has determined something called H, whereas, if one had measured the force on a length of wire carrying a current one would have determined something called B. With a solenoid the assumption that the 'polarity' is concentrated at a point is the end of the make-believe, but with a permanent magnet it is but the beginning, for it is assumed that, if immersed in another medium of different permeability, the pole strength is unchanged; but seeing that 'pole strength' is a fiction which has only been defined in air, it is meaningless to say that it remains unchanged in another medium. When driven to be precise, most physicists would probably admit that they assume the total flux to remain constant, but such an admission undermines the basic nature of the unit pole.

In my opinion, we are concerned in reality with only two forces, namely, that between charges at rest and that between charges in motion, that is, between currents. Just as we write the gravitational formula

$$f = G\frac{M.M'}{L^2}$$

with a dimensional constant, so for the same reason we write (omitting trigonometrical coefficients and signs of integration)

$$f = \frac{1}{K}\cdot\frac{q}{r^2}\cdot q \quad \text{and} \quad f = \mu\cdot\frac{ids}{r^2}\cdot ids.$$

The classical formula $f = mm'/\mu r^2$ is merely the latter formula wrapped in mystery.

These two fundamental formulæ may be written

$$f = \frac{1}{K}\cdot 4\pi D\cdot q = \mathcal{E}\cdot q, \text{ where } \frac{q}{r^2} = 4\pi D \quad \text{and}$$

$$\mathcal{E} = \frac{1}{K}\cdot 4\pi D,$$

and

$$f = \mu\cdot H\cdot ids = B\cdot ids,$$

$$\text{where } \frac{ids}{r^2} = H \text{ and } B = \mu H.$$

We are here confining our attention to space and not considering the numerical effects of a change of medium on K and μ. It is quite a mistake to imagine that the distinction between H and B is necessarily associated with ferromagnetism or with any assumptions as to its nature. The same applies to the distinction between D and $\mathcal{E}$, which is not necessarily associated with phenomena in material dielectrics.

The above formulæ are, of course, linked by the relation $i = dq/dt$, from which it follows that both $\mathcal{E}/B$ and H/D have the dimensions of velocity.

It will be seen from the above formulæ that $\mathcal{E}$ and B are measurable characteristics of the electric and magnetic fields, the former by the force on a stationary charge and the latter by the force on a current or moving charge, irrespective of the medium. D and H, however, are not directly measurable concepts. The displacement D is calculable at any point by dividing a charge by the square of a distance, that is, by an area; and similarly the magnetizing force H is calculable by dividing a current by a length. The latter is often expressed in ampere-turns per centimetre and is regarded as a localized cause producing at every point a magnetic induction B depending on the medium. *Similarly D appears as a calculated localized cause producing at every point a condition of space designated by $\mathcal{E}$ depending on the medium;* this is a reversal of the usual conception.

Some readers may object to the time-honoured magnetizing force H being relegated to the realms of unmeasurable concepts, and may quote from Prof. F. A. Lindemann's recent Guthrie Lecture (*Proc. Phys. Soc.*, 823; Nov. 1, 1936) that "a concept is meaningless unless it is in principle possible to observe the quantity which it typifies", but since both current and length will presumably be conceded as observable, their quotient can scarcely be dismissed as meaningless.

The procedure is analogous to that followed in the theory of elasticity and indeed throughout the whole of mechanical engineering. A load, that is, a force, is applied to a member, say, a tensile test specimen, and produces a strain or displacement at every point. One conceives this strain to be due to the stress at the point, this stress being an unmeasurable concept having the dimensions of a force divided by an area. Just as the mechanical engineer pictures the strain at a point as being due to a localized cause which he calls stress, so the electrical engineer pictures the magnetic condition at a point which he calls the magnetic induction B as being due to a localized cause which he calls the magnetizing force H. As the area integral of the stress is equal to the total applied force, so the line integral of the magnetizing force is equal to the total applied magneto-motive force or ampere-turns (neglecting numerical constants). If any reader feels disinclined to agree that mechanical stress is an unmeasurable concept I would refer him to p. 114 of Prof. R. V. Southwell's recently published "Theory of Elasticity", where he says:

"External loads can be measured, and the displacements of points on the surface of a body (in the case of transparent materials we can make some attempt to measure the displacements of internal points); but *stress* (or internal action) has never been measured directly, and we can assert with some confidence that it never will. In these circumstances all that we can do is to construct a theory on assumptions which are consistent with experimental observations, and to look for confirmation of that theory to tests of particular conclusions—namely, predictions regarding displacements which can be measured."

It is to be noted that the one circuital equation

$$\frac{d\Phi}{dt} = A\,\frac{dB}{dt} = E = \mathcal{E} \,.\, l$$

involves only the directly measurable concepts, whilst the other

$$4\pi A\,\frac{dD}{dt} = m.m.f = H \,.\, l$$

involves only the non-measurable concepts.

On p. 299 of his book, Dr. Lanchester says, "In the opinion of the author it [μ_0 the permeability of space] is a mere number, a numeric". We can only say that we have failed to find anywhere in the book any valid reason, other than convenience, for this assumption, and that we have set out the above formulæ with the intention of showing clearly that there is no more reason for regarding μ_0 as a numeric than there is for making this assumption with regard to K_0. If, however, one assumes either explicitly or implicitly that a permanent magnet pole measures H, whilst a current-carrying conductor measures B, and then assumes that a permanent magnet is a conglomeration of molecular currents, it is little wonder that when one puts $B = \mu H$, one is able to prove that μ is merely a numeric.

In Appendix V, Sir James Henderson quotes Maxwell in support of the view that μ_0 is a dimensionless numeric, but the support is very unconvincing and one sentence is definitely against this view. The author attempts to brush this aside with the suggestion that Maxwell was nodding when he wrote the sentence, and he even goes so far as to rewrite it for him—to "re-mould it nearer to the Heart's Desire". We prefer to adopt a humbler role and to suggest that the quotation is by no means convincing that by "identical" Maxwell meant dimensionally and not merely numerically identical in the electromagnetic system of units, as the sentence which he dislikes certainly suggests. When Maxwell appears to support his view his "vision was wonderful"; when the support appears doubtful Maxwell's "clear view was obscured".

In Appendix VII Dr. Lanchester disagrees with Giorgi and the S.U.N. Commission of the International Union of Pure and Applied Physics and concludes that "the M.K.S. or Giorgi system requires that the new unit of μ_0 shall be 10^7 c.g.s. units and not 10^{-7} as given in the various reports and memoranda". μ_0 is not a symbol for permeability but for one special permeability, namely, that of space, and we do not know what the author means by "the new unit of μ_0". He seems to have been confused by the unfortunate wording of the Report of the S.U.N. Commission, which stated that "the 'fourth unit' of the M.K.S. system is 10^{-7} henry per metre, the value assigned on that system to the permeability of space". This is very misleading. In the M.K.S. Giorgi system in which the absolute units of current, resistance, etc., are the ampere, ohm, etc., a medium to have unit permeability must be 10^7 times as permeable as space, and therefore the permeability of space, μ_0, is 10^{-7}, if expressed in M.K.S. Giorgi units. A further source of confusion is probably the introduction of the unit of inductance, the henry, into the definition, for although the dimensions of inductance are those of permeability multiplied by a length, its numerical value involves such things as the number of turns, and we have always regarded its inclusion in the Report as meaningless. It is not surprising that Dr. Lanchester has found it very confusing.

How much trouble has been caused by the persistent attempts to force by hook or by crook the electromagnetic concepts into the framework of L, M and T? Nothing in this book is more reasonable than the concluding paragraph of Rücker's paper, namely, that by not suppressing the secondary fundamental units such as μ and K,

"I think that the symbols are thus made to express the limits of our knowledge and ignorance on the subject more exactly than if we arbitrarily assume that some one of the quantities involved is an abstract number."

The Giorgi suggestion to regard one of the electromagnetic concepts as fundamental is in keeping with this. If permeability be thus adopted as fundamental, there can be no question of its dimensions. The dimensions of length, mass, time and permeability are then L, M, T and 'μ', and just as certain values of the first three (C.G.S. or M.K.S.) are taken as units, so in the Giorgi system, unit permeability is 10^7 times that of space. All the electromagnetic concepts can be expressed dimensionally in terms of L, M, T and 'μ' without ambiguity and without making arbitrary and unjustifiable assumptions. If, in the future, Nature divulges some secret whereby the limits of our knowledge are extended so that we are able to express 'μ' in terms of L, M and T, the dimensional expressions can then be readily reduced to these three fundamentals; but in the meantime let us be honest with ourselves and not pretend to knowledge that we do not possess.

In Appendix VI Dr. Lanchester attempts to obtain light on the subject by calling in relativity, but seeing that it leads him to the conclusion that "a wave generated by a static charge undergoing acceleration will, in free space, be a wave having only magnetic characteristics, but which when arrested will revert to the static form", whereas "radiation from a magnetic pole or doublet would be wholly static in its manifestations until received and brought to rest", I am forced to conclude that in his search for light Dr. Lanchester has wandered into the outer darkness. In my opinion the author was unwise to include this section in a book the avowed object of which is to help the young engineer.

G. W. O. HOWE.

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(Berlin: Verlag Chemie, G.m.b.H., 1935–36.)

(1, 2 and 3) THE various hydrides and oxides of nitrogen comprise the subject matter of parts 2 and 3 of the volume on nitrogen, but aqueous solutions of ammonia and the oxy-acids are not included. Prominence is given to the preparation of the raw materials used in the industrial synthesis of ammonia. It is pointed out that nitrogen extracted from air is very much cheaper than hydrogen, which comes chiefly from water-gas (51·8 per cent), coke-oven gas (30 per cent) and electrolytic plants (16·67 per cent). The Linde, Claude and Messer processes for extracting hydrogen from coke-oven gas are given very fully, as well as the synthesis of ammonia by the Haber-Bosch, Claude, Fauser and Mont Cenis methods. There is also an account of heavy ammonia, ND_3, which has been obtained pure by the action of deuterium oxide on magnesium nitride.

The action of different types of electric discharge upon air and other mixtures of oxygen and nitrogen is fully discussed, but the well-known processes of Birkeland-Eyde and Pauling are not described, since it is reported that they have been largely superseded and indeed contribute at present less than one half per cent of the world's production of synthetic fixed nitrogen. There is, however, a long list of publications upon the processes.

The fourth section of the volume on nitrogen contains an account of the oxyacids and their derivatives. It is stated that nitrous acid is still unknown in the free state, but its importance has attracted the attention of many investigators. Of the three possible structural formulæ, that one