Letters to the Editor

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Refraction of Ionised Media

In a recent letter Prof. Hartree[1] has directed attention to certain difficulties in the theory of refraction as applied to ionised media, and has criticised previous letters by Tonks[2] on the subject. I have myself been occupied for some time in trying to clear up this matter, and can confirm Hartree's opinion of the subtlety of the subject, but at the same time it should be said that, in spite of the defective proof, Tonks's result is certainly right. The problem is whether the formula of Sellmeier or that of Lorentz should be applied for a gas composed of free electrons moving among ionised atoms, which may be taken as fixed protons without losing the point of the question. The refractive index n is to be derived from atomic characters, and the problem is whether it is

$$S=n^2-1, \quad (1)$$

or

$$L=3\,(n^2-1)\,/\,(n^2+2) \quad (2)$$

which is directly related to these. The alternatives for the ionosphere are whether it is S or L that is equated to

$$-Ne^2\,/\,\pi m\nu^2 \quad (3)$$

where N is the number of electrons per c.c., ν the frequency of incident waves, and e and m, the charge and mass of the electron respectively. If the formulæ are used to estimate the actual electron density of the ionosphere, there is a discrepancy of 50 per cent according to which of them is adopted; so that the question is by no means trivial. The same problem arises with even greater force in connexion with the optics of metals. It has been discussed by Kronig and Groenewold[3]; their defence of the use of S is open to exactly the same criticism as that of Tonks, but from the known values of the optical constants of metals it is here even more certain that S is the correct form.

The essential point of the question consists in making the correct allowance for the mutual forces between the various particles concerned in scattering the light. The question is one in which we do not anticipate any great difference between classical and quantum theory, and it is easier to work with the classical; in the quantum theory of metals it has been usual to consider what is in effect only a single electron, and this cannot possibly throw any light on the present question. The main difficulty lies in estimating the large influence on each electron of its close neighbours, both protons and electrons. Tonks tries to overcome this difficulty by replacing the protons by a uniform distribution of positive charge-density, but this replacement is the crucial point of the problem; it is only done by an illegitimate inversion of the order of integrations, and this inversion leads to a large change in the resulting value. Unrigorous processes, like the inversion of integrations, are so habitually done in physics with impunity, that one is apt to trust them completely; with an unrigorous formulation of the present problem it is easy to find entirely plausible arguments leading either towards L or S. It is quite easy to show that a set of electrons moving in a uniform positive medium will obey a formula in S, and everyone agrees on this; the whole difficulty is to justify the replacement of the protons by the continuum, for there is little resemblance between the smooth motion of an electron in the continuum, and the zigzag path among the protons.

The technical problem of discussing with rigour the optics of a finite volume of any material is formidable, for it demands retarded potentials for the mutual forces of each pair of electrons, and so the system cannot be taken as a self-contained dynamical system, but must be treated with the help of Lorentz's device of making a fictitious spherical cavity round each electron. Most of the difficulty can, however, be avoided by the device of taking a small isolated sphere of the material and calculating the light it will scatter to a distance. If the radius a is much smaller than the wave-length of the incident light, there is no need to allow for retardation and the whole sphere can be regarded as a single dynamical system. A simple optical calculation shows that under incident light of amplitude A, it scatters light as though having electric moment

$$A\,a^3\,(n^2-1)\,/\,(n^2+2),$$

and therefore n is found if we can calculate the moment directly. For a set of separate elastically bound electrons, as in neutral atoms, there results at once a formula in L; at the opposite extreme, with a continuum of positive electricity, an equation is easily formed for the electric moment which leads to a formula in S. The important question is how the moment behaves for a set of discrete protons lying arbitrarily throughout the sphere. This is not the place to discuss details, but it can be seen that the average motion will satisfy the same equation as in the case of a continuum, provided that each electron undergoes many collisions during the period of the light. This condition is satisfied in the ionosphere for the long waves used, and in metals for ordinary light, so that S is the right expression in these cases.

As a general comment, it seems natural that S rather than L should be the more fundamental formula. Lorentz derived L by introducing a spherical cavity that was quite fictitious, and yet the algebraic form of (2) shows clear evidence of a real sphere. The reason is that there is a genuine sphere (or perhaps some other shape arbitrarily orientated) round each molecule; this is its own surface which prevents the entry of other molecules, and it is the existence of this small real sphere, and not the comparatively large fictitious sphere of Lorentz, that is responsible for (2). S is the natural formula for pure electromagnetic systems, and L is an expression of the fact that the systems to which it applies are governed by a law that is not electromagnetic—the exclusion principle which prevents one atom from penetrating another.

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Dec. 28.

[1] NATURE, 132, 929, Dec. 16, 1933.
[2] NATURE, 132, 101, July 15, and 710, Nov. 4. See also a letter by Norton, *ibid.*, p. 676, which seems open to the same criticisms, though his method is not very fully described.
[3] *Proc. Amst. Akad.*, 30, 974; 1932.