regular polygons was made until the time of Gauss, who, at the end of the eighteenth century, showed that the construction of a regular p^m-sided polygon, p being prime, depends on a succession of algebraic equations the degrees of which are the prime factors of $p^{m-1}(p-1)$. Any quadratic equation being solved by the intersections of a line and a circle, it follows that a regular polygon of 2^m+1 sides, when this number is prime, can be constructed by Euclidean geometry. The chain of quadratic equations leading to the value of $\cos(2\pi/17)$ in radicals was fully discussed by Gauss ("Disq. Arith.," § 354). Various writers have dealt with the geometrical construction of a regular 17-gon, and it seems unlikely that any improvement in Richmond's construction (Hudson's "Ruler and Compasses," p. 34) will be possible. Richelot (*Crelle's Journal*, Bd. 9) dealt with the sequence of seven quadratic equations which give $\cos(2\pi/257)$ in radicals, and the same problem has been treated geometrically by Affolter and Pascal (*Napoli Rendiconti*, 1887). Euclid's argument which leads to the construction of a regular quindecagon can be extended to apply to regular polygons of 680 and $255\times256\times257$ sides, so, in theoretical discussions, it is sufficient to consider regular polygons of p^m sides.

There are at least three methods of approximately constructing a regular polygon of any number of sides by Euclidean geometry. First, since

$$\frac{1}{n}=\frac{a_1}{2}+\frac{a_2}{2^2}+\ .\ .\ .\ +\frac{a_m}{2^m}+R_m,$$

where $a_r=0$ or 1 and $0\angle R_m\angle 2^{-m}$, an angle $2\pi/n$ can be constructed by m repeated bisections of a right angle with an error less than $\pi/2^{m+1}$. Bisection of very small angles can be avoided by a rearrangement of the expansion for $1/n$.

The second method depends on the formula

$$b^2+c^2-a^2=2bc\cos A,$$

or an equivalent one, in which appropriate rational values of a, b, c render $A\sim2s\pi/n$ as small as may be desired. Until data are available an element of chance enters into the application of this process, there being no rule for obtaining *small* rational values of a, b, c to give A within a prescribed limit of error of $2s\pi/n$.

In the third method a diameter AOB of a circle, radius r, is divided into $n-1$ equal parts at points of which K is the second from B. Then C being a point on the perpendicular diameter such that $OC=\frac{1}{2}\pi r$, CK cuts the circle at Q (beyond the diameter) and $\angle BOQ=\alpha=2\pi/n$ with an error of the order $1/n^2$. A more accurate construction is given by

$$KB=4r/(n-\tfrac{1}{2}\pi^2+4)=4r/(n-0{\cdot}93),$$

the discrepancy in α then being comparable with $1/n^3$. (A curious but widespread fallacy, of which it would be interesting to know the origin, is that, in the same figure,

$$OC=r\sqrt{3},\quad OK=r(n-4)/n.$$

This rule does not even give a good approximation to a regular figure, the error in α being about 10 per cent. when n is large. A much better approximation, avoiding the introduction of π, is given by

$$OC=\tfrac{1}{2}(\sqrt{2}+\sqrt{3})r,\quad OK=r(n-5)/(n-1),$$

this reducing the error in α to $\frac{1}{6}$ per cent. for sufficiently large values of n.)

When n is such that the ruler and compasses allowed by Euclid fail, the vertices of a regular n-agon inscribed in a circle can always be obtained as the intersections of algebraic curves with the circle. The curve used to deal with an algebraic equation of degree q, a prime factor of $p^{m-1}(p-1)$, should be one which can be constructed by some simple mechanical means and crosses a curve already drawn in q new points. A cubic equation being always soluble by the intersections of a conic section with a circle, one conic, combined with ruler-compasses geometry, is sufficient when $n=7, 9, 13, 97, 193$, whereas two conics are needed for $n=19, 27, 37, 73$ and three when $n=81, 109$. A suitable conic is an ellipse the foci and major axis of which can be constructed by Euclidean geometry, the usual string and pins process then being available.

Prof. Alexander, in the pamphlet now before us, attempts to construct a regular heptagon, and a reviewer must examine what success has attended his effort. In the first place, his constructions, all of which require intricate figures, are approximate: a more accurate approximation to $2\pi/7$ could certainly be obtained by drawing a triangle with suitable rational sides a, b, c. Secondly, one construction involves the use of a point P on an ellipse such that $\angle PAA'=2\angle PA'A$, A and A' being the foci: this point P is not defined as the intersection of the ellipse with a line or a circle. Thirdly, in the most accurate construction, two ellipses are needed to give an approximate value of the angle of a regular heptagon, whereas one ellipse combined with ruler and compasses should give the vertices of the heptagon exactly. So, for three distinct reasons, we are unable to accept Prof. Alexander's constructions as the final word on the question of the regular heptagon.

W. E. H. B.

The Malthusian Principle and Sociology.[1]

THE variations of the birth rates and death rates in the different countries are most easily studied from the diagrams made of them by Dr. C. V. Drysdale, the president of the Malthusian League. Birth rates remained high, except in France and Ireland, until 1876. Then the birth rate of England, Belgium, Holland, Germany, Austria, and Switzerland, and of Europe as a whole, suddenly began to fall. This sudden onset of declining fertility must have been due to contraception. The death rates fell with the birth rates, and in such close correspondence as to show that the fall of the death rates was closely related to the fall of the birth rates. That the progress in medical science and sanitation reduced the death rates in such close parallelism with the birth rates can scarcely have been a coincidence. Moreover, when one examines the vital statistics of Rumania, Chile, and Jamaica, and of Russia from 1881 to 1901, and of Ireland from 1886 to 1911, one finds a horizontally-oscillating or fairly stationary birth rate and a similar death rate. Why were not medicine and sanitation reducing the death rate of any of these countries when their birth rate was stationary? Furthermore, Bulgaria, Ceylon, and Japan had a rising birth rate and a rising death rate from 1881 to 1911. Why did these countries then have a rising death rate notwithstanding medical progress?

Evidently, as Malthus believed, the populations of all the long-settled countries were pressing on their means of subsistence—that is, were not increasing their food supplies fast enough for their needs. For example, consider England. In 1876 the birth rate and death rate were still very high and the average duration of life only about thirty-five years. Then the birth rate fell or new mouths were added more

[1] Abstract of a paper read by Mr. Binnie Dunlop, M.B., on February 23, to the Sociological Society.

slowly, and there steadily declined the proportion of persons dying prematurely from diseases consequent upon under-nutrition; thus the death rate fell steadily and the average duration of life rose. The under-nourished are doomed to premature death no matter how efficient and active the health services may be; nothing can make a population increase faster than the food supply; if there be ten additional persons, and only food for eight, two must soon die. The high correlation between the birth rate and the death rate continued up to 1913; therefore, food shortage was still the cause of what was excessive in the death rate up to 1913. Thus the whole of the fall of the death rate from 1876 to 1913, the big reduction of poverty, and the rise in the average duration of life from about thirty-five years to more than fifty years, must be attributed to the decline of the birth rate steadily reducing the amount of the food shortage.

In all the other old countries, and not excluding France, one may similarly infer that up to 1913, notwithstanding the extent to which the birth rate had fallen in some of them, the population was still pressing on the means of subsistence. The main conclusion is most encouraging and is as follows: Virtually the sole economic difficulty which besets man is that he can only increase his food supply very slowly; consequently, small families and willing work would eliminate poverty.

University and Educational Intelligence.

CAMBRIDGE.—The Allen Scholarship has been awarded to Ralph Cooper, of Trinity College.

The list of those who have recently passed the qualifying examination (in elementary mathematics and mechanics) for the Mechanical Sciences Tripos shows that twenty-three per cent. of those who were successful had not yet come into residence. The curriculum for this examination can usually be dealt with at school, and it is of some satisfaction to realise that the slow but steady rise in the standard attained in the schools is gradually relieving the university of the load of elementary teaching, for which it is often not too well equipped, and with which it really ought not to be encumbered.

Sir Alfred and Lady Yarrow have offered to endow an Eric Yarrow lectureship for the study of Assyriology.

Dr. D. Stockdale, of King's College, has been elected to a fellowship at that College. He has taken both parts of the Natural Science Tripos and was elected to a Senior "1851 Exhibition" studentship in 1923.

MANCHESTER.—The following have been recommended for the honorary degree of D.Sc.: Mr. S. L. Pearce, Electricity Commissioner, formerly Consulting and Chief Engineer and Manager, Electricity Department, Manchester Corporation; Prof. A. C. Seward, Vice-Chancellor of the University of Cambridge, professor of botany there; Dr. A. E. H. Tutton, formerly H.M. Inspector of Schools (Technological Branch), Board of Education.

THE British American Tobacco Co., Ltd., has sent a donation of 105*l.* to the funds being collected for the development of the Imperial College of Tropical Agriculture, Trinidad. Other recent donations and promises include: 250*l.* from The Manchester Cotton Association, Ltd.; 105*l.* each from the Royal Mail Steam Packet Co., Ltd., and Messrs. Alfred Holt and Co.; and 100*l.* each from Lord Queenborough, M.P., and C. H. Gray, Esq. These donations, together with the sum of 15,000*l.* recently granted by the Imperial Government, will be devoted to the building of a hostel for the students, and the provision of an estate where the business side of farming may be taught. A further 25,000*l.* is, however, still urgently required. Contributions may be sent to Mr. Algernon Aspinall, Secretary, 14 Trinity Square, London, E.C.3, or to Barclays' Bank (Dominion, Colonial, and Overseas), 30 Gracechurch Street, E.C.3.

WIDESPREAD interest will be aroused by the publication of the report of the Departmental Committee on the University of London (H.M.S.O. Cmd. 2612). This Committee was asked to consider the final (1913) report of the Royal Commission on University Education in London, and to make recommendations as to changes in the constitution of the University which may now be desirable. The terms of reference were wide, and the Committee, in its discretion, decided to make no recommendations on the questions of the site of the University and the external examination system. One of the main recommendations is that a council of sixteen should have the final control of finance, if and when the Senate approves of the educational policy implied. Nine of the sixteen are to be University representatives—the chancellor and the chairman of Convocation, the vice-chancellor and six others from the Senate; four will be nominated by the Crown, and two by the London County Council; one will be co-opted. As regards the constitution of the Senate, it is suggested that the present 'nominated third' should be replaced by a collegiate council consisting of the vice-chancellor and the principal of the University, and the heads of the seven chief schools and colleges, together with two representatives of the medical schools, and a few others representing minor schools. The chief function of this council will be the supervision of inter-collegiate affairs. The total membership of the Senate is to be reduced from 56 to 49.

UNIVERSITY COLLEGE, Reading, has now received its Royal Charter conferring upon it the full status of a university. This was announced on Saturday, March 20, by Mr. H. G. Williams, M.P., who congratulated the Lord-Lieutenant of Berkshire, Mr. J. H. Benyon, on becoming the first chancellor. This latest addition to the family, now numbering seven, of twentieth-century universities of England, is descended from "The University Extension College, in conjunction with the Schools of Science and Art, Reading," which was opened in 1892 and incorporated under the Companies Acts in 1896. Of its sister universities the youngest is Bristol, chartered in 1909. The University has an endowment of 285,000*l.*, of which 200,000*l.* came from the Palmer family and 50,000*l.* from Lady Wantage. The first principal was Mr. H. J. (now Sir Halford) Mackinder, then a student on the foundation of Christ Church, Oxford. The present principal, Mr. W. M. Childs, has held office since 1903. There are a strong faculty of agriculture and horticulture, and departments of fine arts, music, and domestic subjects, in addition to faculties of letters and science. The National Institute for Research in Dairying forms part of the University. The number of full-time students as shown in the University Grants Committee's returns for 1923-24 is 578, distributed as follows: Arts 339 (including 287 women), pure science 104, agriculture 135. A cardinal principle of the policy of the College from its earliest years has been to stress the importance of the corporate life of its members. This is reflected in the large proportion (70 per cent.) of its full-time students in halls of residence. In other modern English and in Scottish universities a large majority of the students reside in lodgings or at home.