the spherical formations hanging downwards with clear cut edges. If the photograph be turned upside down the appearance is that of the tops of cumulus clouds as seen from an aeroplane above them.

Just as the billowy tops of cumulus clouds are due to the *ascent* of warm moist air into cooler air above, so the globular formation of the festoon-cloud must be caused by the *descent* of warm moist air into an underlying cooler stratum. This inversion of temperature is generally indicative of bad weather, and this was corroborated by the weather experienced at and after the time the photograph was taken.

WILLIAM J. S. LOCKYER.
Norman Lockyer Observatory,
Sidmouth, South Devon.

The Tides.

THE great importance of the subject is my excuse for troubling you once more, very briefly, regarding it. In NATURE of July 21, I stated that, according to the present tidal theory, the tidal forces, and consequently the tides, would be just the same for a sea-depth of about 4000 miles as for the actual sea-depth of about 2 miles; and, in the same issue, your reviewer, "The Writer of the Note," agrees that this is true, or, in his own words, "that the differential motion of the oceans is determined by the vectorial excess of the forces at the earth's surface over those at its centre"; which appears to ignore entirely the depth of the ocean as a factor determining the height of the tides.

The theoretical cause of the tides is the difference of the attractions of thesun and moon at the earth's surface and centre. This difference in the case of the moon is more than twice as great as in the case of the sun; therefore, the lunar tide is more than twice as great as the solar tide. Similarly, if the earth were expanded into a hollow, spherical crust of ten times its present diameter, with its water-covered surface nearest to the moon at the same distance as now, and the moon's period of revolution also remaining the same, then the lunar tide-raising force, and consequently the tide, would be about twelve times as great as now. This is the teaching of the present tidal theory; but is it the teaching of practical mechanics and common sense? Why should the mere expansion of the earth cause a ten, or twenty, or a hundred times greater tide upon its surface, the distance of that surface from the moon, as well as the masses of the earth and moon, remaining the same as before the expansion?

Surely this is a question well worthy of discussion; and surely some of your readers are sufficiently interested and open-minded to express some opinion or argument regarding it.

EVAN MCLENNAN.
Corvallis, Oregon, U.S.A., September 3.

MR. MCLENNAN'S words "and consequently the tides" are not in accordance with dynamics and are not implied in the passage he quotes from my previous note. If the earth were all water the direct tide-generating forces within two miles of its surface would be the same as in an ocean of depth only two miles. These tidal forces are usually represented by reference to the "equilibrium tide," that is, by stating what the outer surface of the oceans would be if the water had lost its inertia without losing its gravitational properties. This outer surface would be the same in the two cases mentioned. The necessary continual adjustment of water, however, would be quite different in the two cases; in the first case the water within two miles of the surface would be largely raised and lowered by that beneath, while in the second case the water would move mainly in a horizontal direction. But owing to the actual inertia of the water the outer surface of the ocean would be entirely different in the two cases, so that the accepted theory does not ignore the depth of the ocean as a factor determining the height of the tides.

The expansion of the solid earth, with an increase in water sufficient to conserve the depth of the oceans, would magnify the tides because the excess of the forces at the earth's surface over those at its centre would expand with the earth's radius. Mr. McLennan apparently finds this result of the gravitational theory repugnant to his common sense.

THE WRITER OF THE PREVIOUS NOTES.

Stirling's Theorem.

IN connexion with the recent letters published in NATURE on Stirling's Theorem, I beg to say that in a paper accepted for publication by the Academy of Zagreb on July 13, and now in print, I proved in quite an elementary manner the formula

$$n! = \sqrt{2\pi}\,.\,(n+a)^{n+\frac{1}{2}}\,.\,e^{-(n+a)},$$
$$a = 0{\cdot}2113249 \text{ or } 0{\cdot}7886751,$$

which coincides with the results published by Mr. James Henderson in NATURE of July 21, p. 97, formula (3). The error was found to be of the order of $1/72\sqrt{3}n^2$ of the calculated value, where $1/72\sqrt{3}$ is equal to $0{\cdot}00801875$ in Mr. Henderson's results. The formula may also be written

$$n! = p\left(\frac{n+a}{e}\right)^{n+\frac{1}{2}}$$

and the log p determined once for all. (For $a = 0{\cdot}2113249$, we have log $p = 0{\cdot}5244599$.) The work of calculation is then by no means greater than in using Stirling's or Mr. H. E. Soper's formula though the approximation is far closer. I think the doubt inferred by Mr. G. J. Lidstone in NATURE of August 25, p. 283, on the usefulness of the formulæ under discussion is not valid so far as the present one is concerned. For sufficiently large values of n, depending on the number of decimals of the tables, the result calculated from the above formula is not worse than that furnished by any other more complicated formula.

STANKO HONDL.
Zagreb, Croatia, SHS-State,
October 7.

PROF. HONDL'S simplified form of my best first approximation to the value of $n!$ follows at once from the fact that $(b-c) = \frac{1}{2}$ in my letter in NATURE of July 21. [b is Prof. Hondl's a.] The constant p in $n! = p\left(\frac{n+a}{e}\right)^{n+\frac{1}{2}}$ is $\sqrt{2\pi}e^{(\frac{1}{2}-a)}$

We have now three approximations involving this type of expression where the index of the power is $(n+\frac{1}{2})$:

$$(1) \qquad \sqrt{2\pi}\left(\frac{n+\frac{1}{2}}{e}\right)^{n+\frac{1}{2}} \text{[Soper]},$$

$$(2) \qquad p\left(\frac{n+a}{e}\right)^{n+\frac{1}{2}},$$

$$(3) \qquad \sqrt{2\pi}\left\{\frac{\sqrt{n^2+n+\frac{1}{6}}}{e}\right\}^{n+\frac{1}{2}} \text{[Forsyth]}.$$

It is interesting to note the increase in accuracy as we proceed from (1) to (3). The errors are $1/24n$, $1/125n^2$, and $1/240n^3$ respectively. Of approximations of this type Forsyth's is by far the most accurate, but for logarithmic calculation it is rather more laborious.

JAMES HENDERSON.
Biometric Laboratory,
University College, London.