had not discovered the modern developments of certain functions as series ; nor had they discovered that differentiation and integration are the inverse of one another. There is little trace in Greek geometry of considerations corresponding to the differential calculus ; the only case that seems certain is that of the subtangent property of a spiral which must have been obtained by the consideration of the instantaneous direction of motion, at any point on the curve, of the point describing it. If, as is probable, Apollonius, in his treatise on the *cochlias* or cylindrical helix, dealt with tangents to the curve, he would no doubt determine the direction of the tangent at any point in the same way.

But the Greeks were by no means limited to what they could obtain by direct integration. They were very ingenious in reducing an integration which they could not perform directly to another the result of which was already known. This must have been the method by means of which Dionysodorus found the content of an anchor-ring or tore and Pappus obtained his theorem which anticipated what is known as Guldin's theorem. In the matter of the anchor-ring the Greeks also anticipated Kepler's idea that the content is the same as if the ring be conceived to be *straightened out* and so to become a cylinder. The Method of Archimedes is mostly devoted to the reduction of one integration to another the result of which is known, but is remarkable also as showing how he obtained certain results otherwise proved in his main treatises. The method was a mechanical one of measuring elements of one figure against elements of another, the elements being expressed as parallel straight lines in the case of areas and parallel plane sections in the case of solids. This point of view anticipated Cavalieri. The elements are really infinitesimals, indefinitely narrow strips and indefinitely thin laminæ respectively, though Archimedes does not say so. But Archimedes disarms any criticism that could charge him with using infinitesimals for proving propositions by carefully explaining that the mechanical procedure does not constitute a proof and is only useful as *indicating* the results, which must then, before they are definitely accepted, be proved by geometrical methods, that is, by the method of exhaustion.

The Disappearing Gap in the Spectrum.[1]

By Prof. O. W. Richardson, F.R.S.

II.

TURNING to Fig. 1, B, which is repeated here for convenience of reference, this shows the various outposts where from time to time spectral lines have been located. It will be seen that there is still a considerable gap between 16·35, the limit obtained with the vacuum grating at the L series of aluminium, and so far. If we consider any typical characteristic X-radiation of an element, for example, the K-radiation, it is found to consist of a number of spectral lines which are denoted by the symbols K_α, K_β, K_γ, in order of ascending frequency. In general there are more than three lines, but we shall adopt the symbol K_γ for the line of highest frequency which is observed, and we
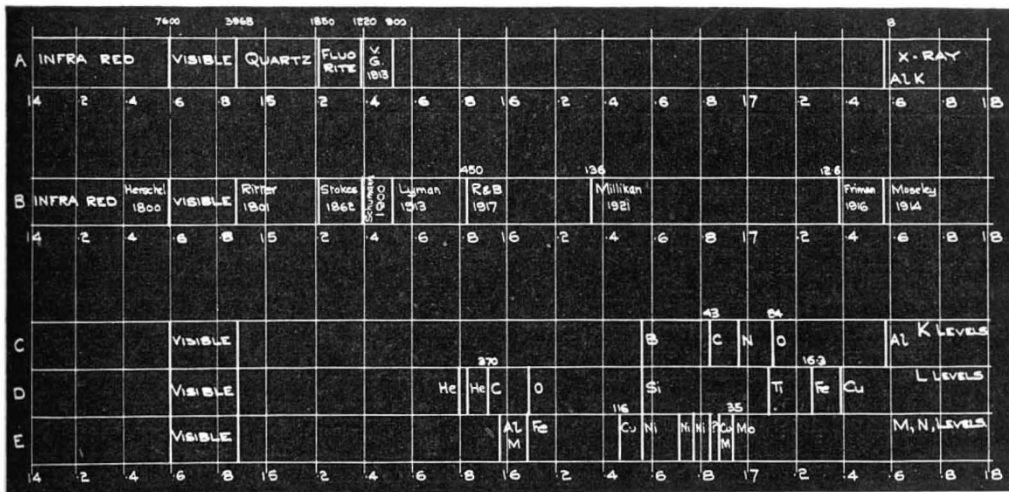

Fig. 1.

17·39, the limit with the crystal spectrometer at the L series of zinc. Between these limits no spectral lines are known, but there is evidence of the excitation of such lines, and data have been obtained for the high-frequency limits of spectra in this region.

This evidence depends upon considerations of a somewhat different character from those dealt with

shall denote its frequency by ν_{K_γ}. These lines can be excited in an element by a stream either of high-frequency radiation or of high-velocity electrons reaching it. In either case the lines are not excited separately, but the whole group K_α-K_γ appears simultaneously. It is found that there are simple and important restrictions on the radiation frequencies and on the electron energies which are capable of

[1] Continued from p. 121.

exciting these lines. Thus it is found that there is a critical radiation frequency ν_c, which is very nearly equal to, but just greater than, ν_{κ_γ}, and unless the incident radiation stream contains components the frequencies of which are at least as great as ν_c, the K series will not be excited. There is a precisely analogous limitation on the electron energies which cause the generation of the characteristic radiations. Thus there is a critical electron energy eV_c, where V_c denotes the critical potential difference through which the electron of charge e has to fall in order to gain this energy, which is connected with the critical frequency ν_c by the quantum relation $eV_c = h\nu_c$, and if the energy of the impinging electrons is equal to, or greater than, eV_c, the characteristic radiations will be excited, otherwise they will not. Furthermore, if we measure the absorption of radiations of different frequencies by the element under consideration, we find that, correspond-
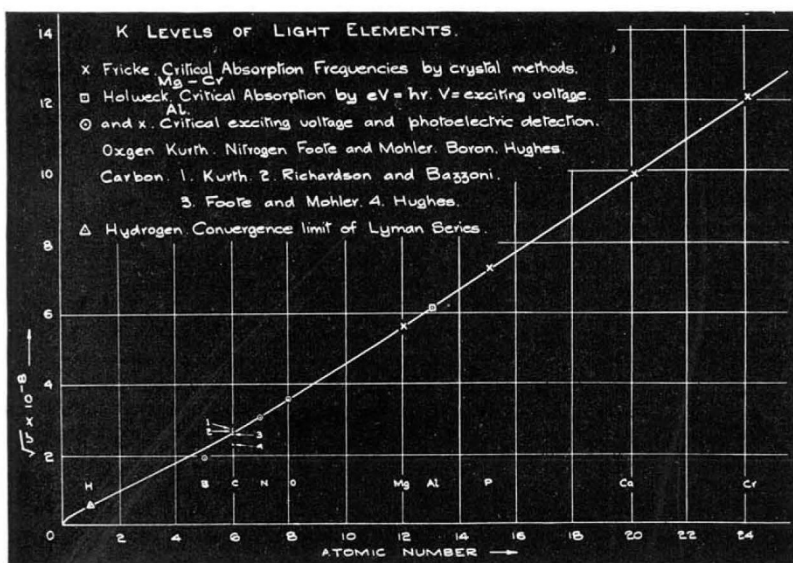

FIG. 4.

ing to the excitation of the characteristic rays, there is a sudden increase in absorption at the critical frequency ν_c. There is also a discontinuity in the ionisation of the element at the same frequency.

There is definite evidence from X-ray phenomena that the critical energy eV_c measures the work which has to be done in removing an electron from its position in the normal atom to a point outside the atom. The characteristic rays are emitted when the gap thus created is subsequently filled up, the different lines arising according to the origin of the electron which fills the gap. If, measured in terms of energy, it is from a near location, we get a low-frequency line such as $K\alpha$; if it is from a location near the surface of the atom, a high-frequency line such as K_γ arises.

Thus critical energies such as eV_c give a direct measure, in terms of energy, of the *levels* of the different electrons in the atom. Alternatively, the corresponding critical frequencies ν_c are the *limits* of the relevant X-ray spectra. If we can determine these limits we shall have found the high-frequency ends of the various spectra. While these ends are not, strictly speaking,

spectral lines, for the heavier elements at any rate, they are very close to the highest-frequency emission lines in the spectra. Furthermore, according to modern spectroscopic theory, they give us the fundamental data on which the formulæ for the spectral series are based.

It is a curious fact that evidence of the existence of such levels in the gap between what are ordinarily termed the X-ray and the ultra-violet spectra should have been produced independently and almost simultaneously by a number of investigators scattered all over the world. These include Foote and Mohler in Washington, Holtsmark in Christiania, Holweck in Paris, Hughes in Kingston, Ontario, Kurth in Princeton, and myself and Bazzoni in London. While the details of the apparatus used by the different workers vary considerably, the principle involved in most of them can be made clear by reference to Fig. 2 (p. 119). Let the element under test forming the anode A_1 be bombarded by a powerful electron current from the hot cathode F. Then any radiation generated at A_1 can pass between the charged plate condenser P, where any ions present will be removed from it, into the chamber on the left. If the rather complicated apparatus shown in the left-hand chamber is removed and replaced by a plate on which the radiation falls and by a second electrode, the radiation can be detected by the photoelectric electron emission it produces at the plate and measured by the current which flows between the two electrodes, the plate being negatively charged. Let this current be measured and divided by the thermionic bombarding current for a series of different potentials applied between F and A_1; then if there is a sudden generation of characteristic rays from A_1 at some critical potential V_c we should expect an increased rate of rise of the photoelectric current with applied potential to set in at V_c. Thus, briefly stated, the experimental method is to plot photoelectric current per unit thermionic current against primary bombarding potential and to look for discontinuities in the resulting diagram. These discontinuities should occur at the critical potential differences V_c corresponding to the energy levels eV_c and to the frequency limits ν_c equal to eV_c/h.

This general type of method leaves much to be desired, but it seems the most practicable procedure at the present stage of the subject. It is open to the general objection that discontinuities in functional diagrams are often merely indications of faulty experimenting, and the evidence that such discontinuities as are observed are really due to the excitation of X-rays is quite indirect and inferential. It is hoped later, however, to make good this deficiency by supplying a direct test of the frequencies of the radiations generated; for example, by using the magnetic spectroscope which was used for determining the end of the helium spectrum, and by other methods.

Fig. 4 shows the square roots of the critical fre-

quencies of the light elements for K-radiations plotted as ordinates against the atomic numbers as abscissæ. The values for all the elements from magnesium to chromium which are amenable to crystal methods have been determined accurately with crystal gratings by Fricke, who measured the wave-length at the absorption discontinuity. They all lie on a curve which is almost a straight line through the origin, and a few of them are shown thus, x. The aluminium value ⊡ is practically identical with Fricke's for the same element and was obtained by Holweck by measuring the voltage V_c on an X-ray tube for which the absorbability in aluminium of the total radiation is a maximum. This method contains features which, though found separately in the method used by Fricke and in the photoelectric methods, are not common to both, and the agreement will no doubt tend to promote confidence in the photoelectric methods. The points for oxygen (Kurth), nitrogen (Foote and Mohler), carbon (Foote and Mohler, Hughes, Kurth, Richardson and Bazzoni), and boron (Hughes) have all been obtained by photoelectric methods. The hydrogen point △ is the limit of the Lyman series which should correspond to the K level for hydrogen. It will be seen that the hydrogen, nitrogen, and oxygen points practically fall on a smooth curve which is continuous with the curve for the elements from magnesium to chromium. There is some disagreement in the case of carbon, but three of the points are very close to the same curve. The only notable deviation is the low value given by Hughes. The boron value also falls below this curve but there is, so far as I am aware, no known reason why the frequencies should be a smooth function of atomic number for these very light elements.

The next lower critical frequency for any element will presumably be that pertaining to the L group, or the highest L critical frequency if there is more than one. The square roots of a number of such critical frequencies for elements from boron to copper as given by photoelectric methods (boron and carbon, Hughes; carbon, oxygen, aluminium, silicon, titanium, iron, and copper, Kurth) are shown thus, x, in Fig. 3 (p. 120). These frequencies should be somewhat higher than those of the corresponding lines, and it will be seen that the observed points from aluminium to copper are all about the same distance above the broken projection of the curve through the values for the L_{a_1} lines for the elements from zinc to zirconium obtained by crystal measurements. This affords additional justification for extrapolating from the zirconium to zinc L_{a_1} values to the value for the L_a line for aluminium as was done in interpreting Millikan's vacuum grating data. It will also be observed that the values of the limits for boron, carbon, and oxygen given by the photoelectric methods are either very close to the values for the shortest lines in the L spectra found by Millikan or have a somewhat higher frequency. These properties are in harmony with those found in what is more usually regarded as the X-ray region. It should be added that data for elements between sodium and chlorine have been given by Mohler and Foote, which fall on or below the L_a curve as drawn in Fig. 3. These data, however, have been obtained by the electron bombardment of vapours, in many cases of compound vapours, and it is not improbable that the values for these will be different from those for the solid elements. Some of these data also appear to refer to radiation potentials, which correspond to lines, rather than to ionisation potentials, which correspond to limits.

Just as in the case of the L_a lines, the L limits for the light elements from helium to magnesium do not change smoothly with increasing atomic number as do the limits for the heavier elements. In fact the frequency for helium as obtained either by direct determination of the end of the corresponding spectrum or from the ionising potential is higher than that of succeeding elements until carbon is reached.

In the case of a number of elements ranging from aluminium to molybdenum, critical potentials have been observed (by Kurth and by Richardson and Bazzoni) at values corresponding to frequencies well below those which characterise the L spectra. The connexion with the generally recognised X-ray series of the heavier elements has scarcely yet been worked out in sufficient detail for the precise group allocation of some of these to be determined with certainty.

Turning to Fig. 1, C, D, and E show, on the same scale as in A and B, the position of some of the spectral limits given by these photoelectric methods. It will be seen that a majority of them lie in the gap between 16·35 and 17·38 in which so far no spectral lines, either X-ray or ultra-violet, have been detected by grating methods. If the interpretation of these photoelectric determinations as the ends of the various spectra is substantiated, it will have to be admitted that the gap in the spectrum between the ultra-violet and the X-ray region about which I have been speaking is not merely disappearing but has actually disappeared.

Obituary.

PROF. JOHANNES ORTH.

PROF. JOHANNES ORTH, whose death is announced, was born in 1847 at Wallmerod in Nassau. He received his medical and scientific training chiefly at Bonn, where he studied pathology under Rindfleisch, whose assistant he afterwards became. Later, he was appointed assistant to Virchow in Berlin. In 1878 he was appointed professor of general pathology and pathological anatomy in Göttingen and afterwards received the title of *Geh. Med.-Rat.* In 1902, on the death of Virchow, he was elected to the chair of pathology in the University of Berlin, and since then his energies have been devoted chiefly to the development of the Institute of Pathology, which was founded and equipped by Virchow.

Orth was the author of numerous papers on pathological subjects, and also of several books, the two most important of which were his " Compendium der pathologisch-anatomischen Diagnostik," which was translated into English in 1878, and his " Lehrbuch der speciellen pathologischen Anatomie," published in 1893. Orth was undoubtedly a pathologist of great eminence and made many valuable contributions to his subject, but his reputation rested rather on his powers as a teacher and expositor and on his width of knowledge