

and use  $r'$  instead of  $r$  as our radial co-ordinate. Whether we use (A) or any other expression, we have to find out from the expression itself the meaning of the co-ordinates introduced. In the limiting case  $m=0$ , the above expression agrees with the formula for polar co-ordinates and time in a Euclidean world; hence it is usual to call  $r$  the distance from the sun and  $t$  the time. But there can be no exact identification of variables in a non-Euclidean world with quantities the definition of which presupposes a Euclidean world; and the only exact definition of  $r$  and  $t$  is that they are mathematical intermediary quantities which satisfy equation (A). The variable  $t$  is in no sense an absolute time; it is specifically associated with the sun, which in equation (A) is regarded as the only mass in the universe worth considering.

Without troubling about the approximate identification of  $t$  with our common notion of time, our results may be stated in the following form:—At a point in the laboratory ( $r=\text{const.}$ ),  $dt_1$  for a light vibration from a solar atom differs from  $dt_2$  for a terrestrial atom. It follows from the formula (A) that  $ds_1$  and  $ds_2$  will differ in the same ratio, since we are now concerned only with the relation of  $dt$  and  $ds$  on the earth. The intermediary quantity  $t$  is thus eliminated; and the difference in the light received from solar and terrestrial sources is an absolute one, which it is hoped the spectroscope will detect.

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**The Straight Path.**

In my book, "A Theory of Time and Space," I directed attention to the fact that in the simple four-dimensional time-space theory there are three types of plane in addition to three types of line.

On p. 360 I stated the following results:

"If A, B, C be the corners of a general triangle all whose sides are segments of one kind, then:

"(1) If the triangle lies in a separation plane, the sum of the lengths of any two sides is greater than that of the third side.

"(2) If the triangle lies in an optical plane, the sum of the lengths of a certain two sides is equal to that of the third side.

"(3) If the triangle lies in an acceleration plane, the sum of the lengths of a certain two sides is less than that of the third side."

These results were published in 1914, and, in spite of the fact that they were printed in italics, so that he who runs might read (that is to say, provided anyone should run on the occasion of reading my book), yet I still find writers continually making statements to the effect that the straight line in this geometry is the shortest distance between its extremities.

As a matter of fact, what I call a "separation line" lies in all three types of plane, and is, consequently, neither a minimum nor a maximum, while an "inertia line" can only lie in acceleration planes, and can easily be seen to be a maximum in the mathematical sense. Further, a triangle cannot have all its sides formed of segments of "optical lines."

I have long contended that the usual method of approach to what is generally called the "theory of relativity" is quite inadequate, and this is a further illustration of my contention.

Not only are our ordinary ideas as to space and time disturbed, but also our ideas of simultaneousness and our notions of "straight lines" in the resulting four-dimensional geometry.

From the midst of this wreckage a logical theory has to be constructed, and the difficulty is to find any firm basis at all.

In the course of my own work I succeeded in finding

what appears to be such a basis in the relations of *before* and *after*.

On this basis I found it possible to construct a theory of time and space (apart from gravitation) which led to the same equations as those of Einstein, but of such a nature as to be independent of the particular observer, and therefore truly physical and devoid of the subjectivity which seems to cling to Einstein's theory.

These relations are, in fact, what might be described as *physical invariants*, and, with the help of certain postulates concerning them, they serve as a basis for a system of geometry.

If this investigation had been published in the German language it would doubtless have attracted more attention on the part of British physicists, who might then have added the ideas of *before* and *after* to their store of fundamental physical concepts. Instead of this, however, I have seen no mention of them at all in recent discussions on the so-called relativity theory. It is true, of course, that no analysis of Einstein's recent work has as yet been made in terms of the relations of *before* and *after* but seeing that these have proved a sufficient basis for the simple theory corresponding to Euclidean space, and that such relations do actually hold in our experience, it does not seem unreasonable to suppose that with modified postulates they might serve as a basis for the more general theory.

With regard, however, to my statement that the straight line in the simple theory is not the shortest distance between its extremities, I can imagine some people casting doubts upon my veracity. For the benefit of those who do not believe me, I venture to give some simple arithmetical examples.

Taking  $v$  as unity, the length  $s$  of the segment of a separation line between elements the co-ordinates of which are  $(x_0, y_0, z_0, t_0)$  and  $(x_1, y_1, z_1, t_1)$  is given by the equation:

$$s^2 = (x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 - (t_1 - t_0)^2.$$

Let A, B, C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub> be elements the co-ordinates of which are as follows:

	$x$	$y$	$z$	$t$
A	0	0	0	0
B	10	0	0	0
C <sub>1</sub>	5	12	0	0
C <sub>2</sub>	5	5	0	5
C <sub>3</sub>	5	0	0	4

On substituting these values we get:

$$\begin{aligned} AB &= 10 \\ AC_1 &= 13 & C_1B &= 13 \\ AC_2 &= 5 & C_2B &= 5 \\ AC_3 &= 3 & C_3B &= 3 \end{aligned}$$

Thus we have:

$$\begin{aligned} AC_1 + C_1B &> AB \\ AC_2 + C_2B &= AB \\ AC_3 + C_3B &< AB \end{aligned}$$

For the case of an inertia line the length  $\bar{s}$  is given by the equation:

$$\bar{s}^2 = (t_1 - t_0)^2 - (x_1 - x_0)^2 - (y_1 - y_0)^2 - (z_1 - z_0)^2.$$

As before, let A, B, C be elements the co-ordinates of which are as follows:

	$x$	$y$	$z$	$t$
A	0	0	0	0
B	0	0	0	10
C	4	0	0	5

$$\begin{aligned} \text{Here } AB &= 10, AC = 3, CB = 3. \\ \text{Thus } AC + CB &< AB. \end{aligned}$$

These examples should be sufficient to give an air of plausibility to my statements.

Cambridge, January 23.

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