with microscopes under ordinary conditions if we want to get the best optical effect. We may, as a matter of convenience, have still higher magnifications, because it is not given to everybody to appreciate fine detail unless an image is somewhat enlarged. But it must be appreciated that any increase beyond 750 or 800 diameters does not result in our seeing anything more. It simply allows us to see the object on a somewhat larger scale. We may, therefore, summarise as follows: An object which is much smaller in size than the resolution limit can be rendered visible provided the light with which it is illuminated is of sufficient intensity and sufficiently different in refractive index from the medium in which it lies. To resolve a series of equidistant points or lines in an object, their distance apart must exceed half a wave-length of light in the medium in which the object is immersed. Johnstone Stoney has shown that a pair of lines or objects can be separated when their distance apart is rather smaller than the resolution limit required for a number of points or lines in a row. But it should be borne in mind even here that the resolution limits apply if a clear standard of definition is required. An isolated object or pair of objects are not so well defined if they exceed the resolution limits as laid down for recurring structures. It cannot be too fully appreciated that illumination is the keynote of all sound microscopic work, and this applies whether the illumination is by means of visible radiation under ordinary conditions of work, or whether it is in experimental work in which the use of invisible radiations are concerned.

There is much room for research in this direction, and it is to be hoped that this is one of the points which will be seriously taken up. Apart from any question of research, the education of the user is perhaps of vital importance. It is of little use for opticians to make great efforts to turn out a satisfactory instrument if the user is incapable of taking advantage of the quality of the optical or other parts. I trust, therefore, that this symposium will give an impetus in this direction, and that it will help microscope-users to realise how much remains to be done.

MICROSCOPICAL OPTICS.[1]

IN the opening paragraphs attention is directed to the methods of treating the aberrations on the principle of equal optical paths (A. E. C., Monthly Notices of R.A.S., January and March, 1904, and April, 1905) and to the author's recent determination of the actual light distribution at and near the focus in the presence of aberration (Monthly Notices, June, 1919). The sine-condition is also discussed.

The origin and effects of the secondary spectrum are then dealt with, and the paper proceeds:

The attempts to produce varieties of *glass* free from this secondary spectrum have been unsuccessful so far as the microscope is concerned, for the existing crowns and flints with proportional dispersion have so little difference in dispersive power that an impracticable number of lenses would have to be used to secure the desired effect. We therefore still depend on the material the value of which for this purpose was discovered by Abbe, the natural mineral fluorite, used instead of crown glass in combination with heavy crown glasses or very *light* flint glasses in place of ordinary *dense* flint glass. It was by the use of fluorite that Abbe produced the apochromatic objectives, and fluorite of good optical quality must be used to this day to secure the result. Apart from the difficulty of finding this material, there is no obstacle to the designing by exact calculation of apochromatic objectives.

I now come to a defect of nearly all microscope objectives, and especially of highly corrected ones, which is well known to all practical microscopists, namely, the pronounced curvature of the field, invariably in the sense of requiring a shortening of the distance from object to lens in order to obtain a sharp focus in the outer parts of the field of view. The general theory of the primary aberrations of oblique pencils shows that any lens system when freed from astigmatism will have the curvature of field defined by the Petzval theorem, and that in the presence of astigmatism the two focal lines which then represent the strongest concentration of the light always lie both on the same side of the Petzval curve and at distances from it which are in the approximate ratio of three to one. When the astigmatism is undercorrected the natural curvature of the field defined by the Petzval equation becomes aggravated, whilst overcorrected astigmatism tends to flatten the field, and is deliberately introduced for this purpose in ordinary photographic objectives. The presence of considerable amounts of astigmatism, of course, renders really sharp marginal images impossible in either case, so that its absence, or, better still, a modest amount of overcorrected astigmatism, must be regarded as the ideal in microscope objectives. Unfortunately, this desirable state cannot be reached in the existing types of objectives. The binary low-power objectives up to the ordinary 1 in. and ⅔ in. come nearest to it, and are, therefore, justly liked by microscopists for all work for which they are sufficiently powerful. In the ordinary ternary objectives of the ⅙-in. type, with approximately plano-convex components, the curvature of the field is also of reasonably moderate amount. But it is a general experience that highly corrected objectives are very much worse as regards curvature of field. In the light of my most recent work on the general theory of lenses (Monthly Notices, November, 1919), this curious and objectionable peculiarity is easily explained, and becomes revealed as a *necessary* consequence of high spherical and chromatic correction if the usual number of components is adhered to. In the Lister and Amici types of ordinary objectives, which are fairly satisfactory as regards curvature of the field, the front lens is of such a form as to produce strong outward coma, and there is in the back lens or lenses a corresponding amount of inward coma.

The simple extensions of Seidel's theory, given in the paper last referred to, show that this is the state of affairs which tends to diminish undercorrected astigmatism, or even to reverse it into the more desirable overcorrected form. High correction of the zonal spherical aberration, and to a still greater extent complete removal of the spherical variation of chromatic correction, necessitate a more or less complete reversal of the coma effects in front and back components. In other words, with the usual types of objectives reductions of curvature and apochromatic or semi-apochromatic correction are completely antagonistic and incompatible; what benefits one correction is detrimental to the other. Fortunately, the extended theory also indicates a way out of this dilemma. It appears fairly certain that by building the objective itself on the lines required by the apochromatic condition, but leaving it spherically undercorrected, perhaps also chromatically overcorrected to a moderate extent, and with a considerable amount of outward coma (this is the most important), and by correcting these residuals in a *widely separated* additional back lens, it will be possible to combine moderate curvature of field with apochromatic perfection, and thus to remove the worst outstanding defect of the best objectives.

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[1] From a paper by Prof. A. E. Conrady presented at a discussion on "The Microscope: Its Design, Construction, and Applications," organised by the Faraday Society at the Royal Society on January 14.

Condensers for the proper well-regulated *illumination* of microscopic objects are identical in optical design with objectives, the only difference being that the light passes through in the reverse direction, and that a lower degree of correction is sufficient not only on theoretical, but also on practical grounds, for nearly always condensers are used in conjunction with the "plane" mirror, which invariably is very far from optical perfection, and so introduces irregular aberrations of unknown magnitude and kind, and, moreover, the light from the condenser has to pass through the slide on which the object is placed. This slide is practically little better than window-glass so far as optical quality and perfection of surfaces are concerned, and the great variation in thickness is another source of imperfection, especially with dry condensers of high N.A.

Moderate amounts of residual aberrations in condensers can always be effectively neutralised by using a sufficiently large source of light of uniform brightness or by magnifying the source by a sufficiently well-corrected "bull's-eye," if the source of light is naturally small.

A great and very serious defect in the construction of nearly all condensers of the present day, with the exception of the modest Abbe condenser of two simple uncorrected lenses, is that the iris and the ring for dark-ground stops are placed too far from the back lens instead of being close to the anterior focal plane of the condenser. It is easily shown that such a remote iris-opening or dark-ground stop produces decidedly oblique illumination of the extra-axial points of the object. With direct light this leads to an undesirable variation in the type of image and in resolving power in different parts of the field. With dark-ground illumination the result is even more serious, for it is then necessary to use a far larger central stop to secure a dark background over the whole field than would suffice if the stop were placed close to the anterior focal plane of the condenser; such an unnecessarily large stop is highly objectionable, because it reduces the visibility of the coarser structures in the object.

The increasingly bad position of the iris in the condensers of higher power and shorter focal length supplies practically the whole explanation of the universal experience that high-power condensers will not work satisfactorily with low-power objectives, especially for dark-ground illumination.

The great thickness of the mechanical stage in English stands of the highest quality is the chief *reason* why the iris and "turn-out ring" of high-power condensers have to be mounted so far below the back lens, and a profound modification of the design of the stage with the view of making the part projecting over the condenser as thin as possible therefore appears to be the most desirable improvement of microscope stands from the optical designer's point of view.

As regards the *actual making of microscope objectives*, it must be borne in mind that the excellence of a computed lens system may be completely swamped by comparatively slight imperfections of workmanship, and that high accuracy in this respect is therefore of the utmost importance. In lenses of high N.A. computation shows that a departure from the prescribed radii and thicknesses by a fraction of a thousandth of an inch may lead to a notable loss of perfection, and the polished surfaces must also be truly spherical within less than half a wave-length of light. These limits can be easily observed if modern methods of gauging and measuring are adopted, and if all surfaces are polished to accurately made and conscientiously used test-plates. The tools and methods employed in really *manufacturing* lenses on this system were shown by Messrs. W. Watson and Sons, Ltd., at the exhibition at King's College in January, 1917, and will be found described and illustrated in the record of that exhibition.

In old English practice the component lenses of microscope objectives and condensers used to be fixed in their cells by cement of the sealing-wax type. Many old lenses which are still found in perfect adjustment fifty or more years after being mounted demonstrate that the cement may hold the lenses in correct position almost indefinitely; but other experiences, especially with lenses used in tropical countries, suggest that shifting may occur, and it is therefore strongly to be urged that all microscope lenses should be held between metallic shoulders at both ends by being bevelled into their cells, care being naturally required to avoid pressure and distortion through too tight a fit.

A point on which users of objectives err to their own detriment is an excess of faith in numerical aperture. I have heard microscopists boast of possessing an objective, say, of 1·43 N.A., whereas somebody else had one of barely 1·40; and a careful test would show that whilst the 1·43 was an indifferent lens, the 1·40 was excellent. The fancied advantage of 2 per cent., then, is really a disadvantage of perhaps 25 per cent. or more.

One of the few disservices which Abbe did to microscopy was the pushing of the N.A. of dry lenses to 0·95, and to a less extent the increase of that of oil lenses to 1·40. The extreme marginal zone of the apochromatic dry objectives of 0·95 N.A. is particularly badly corrected, so much so that the lenses will only bear a solid illuminating cone of about 0·65 N.A. even on the Abbe test-plate, and that with annular light bringing only the marginal zone into action correction-collar and tube-length combined do not allow of reaching a point of good spherical correction. There is no doubt that Abbe's own earlier dictum still holds, to the effect that beyond about 0·85 N.A. the higher aberrations become unmanageable unless the free working distance is reduced to a very few thousandths of an inch. A carefully computed objective of 0·85 N.A. will bear a full illuminating cone on suitable objects, and can thus realise its fullest resolving power. An objective of 0·95 with a condenser of 0·65 has the resolving power of the mean, or of 0·80 N.A., and is thus actually inferior, except for freak resolutions, with extremely oblique light. Oil objectives of more than 1·30, or at most 1·35, N.A. are also of very doubtful added value.

In closing this section I will once more quote without comment an anecdote of Fraunhofer, who received a complaint that a telescope supplied by him, although giving magnificent images, displayed certain fine scratches when examined with a magnifying-glass! The reply sent by Fraunhofer is reported to have been: "We have constructed the telescope to be looked through, not to be looked at."

A few sentences may perhaps be added as to the prospects for further improvements of microscopic performances. I have stated earlier in this paper that there is a bright ray of hope with regard to diminishing the curvature of field without loss of definition.

Advances in numerical aperture offer very little attraction. Abbe, in my opinion, carried the N.A. too far rather than not far enough, and I am not aware that any notable discovery has been achieved with the few monobromide immersion objectives of N.A. 1·60 which he designed.

The use of a shorter wave-length, *i.e.* ultra-violet light, is a little more promising. There would be none but technical difficulties to the construction of lenses suitable for this work. But as only very few microscopists would be likely to go to the trouble of working in invisible light and of passing through a long apprenticeship in mastering the difficulties, apparatus of this description would necessarily be extremely costly, as the whole expense of designing and of constructing special tools would fall on a small number of outfits, or possibly on only a single one. And there would still be the grave drawback that the vast majority of objects would be opaque to extreme ultra-violet rays, and yield only black-and-white outline pictures.

The so-called ultra-microscope does not represent any advance in *resolving* power at all, but most decidedly the reverse. It is highly valuable for the detection of very minute particles and of their movements, which it achieves simply by intense dark-ground illumination, but the structure of the particles remains unrevealed, and only that would amount to an advance in resolving power. The seeing of these minute particles is, in fact, of precisely the same kind as the seeing of stars subtending less than 0·001 second of arc at night with the naked eye, the resolving power of which is of the order of 60 seconds.

PARIS ACADEMY OF SCIENCES.

BONAPARTE AND LOUTREUIL FOUNDATIONS.

OF the 72,500 francs placed at the disposal of the Academy by Prince Bonaparte, it is proposed to allocate 30,000 francs as follows:—

5000 francs to Charles Alluaud, travelling naturalist to the National Natural History Museum, for a geological and botanical expedition in the Moroccan Grand Atlas Chain.

2000 francs to A. Boutaric, for the construction of an apparatus for recording nocturnal radiation.

1000 francs to Emile Brumpt, for continuing his work on parasitic hæmoglobinuria or piroplasmos of cattle.

3000 francs to E. Fauré-Fremiet, for undertaking a series of studies on histogenesis and certain surgical applications.

3000 francs to A. Guilliermond, for pursuing his researches on lower organisms and on mitochondria.

3000 francs to Joseph Martinet, for continuing his researches on the isatins capable of serving as raw material for the synthesis of indigo colouring matters.

3000 francs to A. Vayssières, for the continuation of his researches of the marine molluscs, family Cypræideæ.

10,000 francs to the Fédération française des Sociétés de Sciences naturelles, for the publication of a fauna of France.

The committee appointed to allocate the Loutreuil foundations recommend the following grants:—

(1) To establishments named by the founder:

10,000 francs to the National Museum of Natural History, for the reorganisation of its library.

7500 francs to the Paris Observatory, at the request of the Central Council of the Observatories, for purchasing an instrument.

(2) Grants applied for direct:

6000 francs to the Société Géologique du Nord, to enable it to take up work interrupted by the war.

10,000 francs to l'Ecole des hautes études industrielles et commerciales de Lille, for restoring the material of its chemical laboratory.

20,000 francs to the Observatory of Ksara (near Beyrout). This laboratory was practically destroyed by the Turks and Germans. The grant is towards its restoration.

8000 francs to Henri Deslandres, for the study of the radial movements of the solar vapours and the thickness of the gaseous atmosphere of the sun.

7500 francs to Maurice Hamy, to carry out certain improvements in astronomical apparatus of precision.

3500 francs to Félix Boquet, for the publication of Kepler tables.

1000 francs to G. Raymond, for the continuation of his actinometric experiments.

10,000 francs to Charles Marie, for exceptional expenses connected with the publication of the "Tables annuelles de constants et données numériques de chimie, de physique et de technologie."

10,000 francs to the Fédération française des Sociétés de Sciences naturelles, for the publication of a French fauna.

2000 francs to P. Lesne, for his researches on the insects of peat-bogs.

2000 francs to A. Paillot, for his researches on the microbial diseases of insects.

2000 francs to Just Aumiot, for the methodical study of the varieties of potato.

5000 francs to Albert Peyron and Gabriel Petit, for the experimental study of cancer in the larger mammals.

3000 francs to Th. Nogier, for completing the installation of the radio-physiological laboratory of the Bacteriological Institute of Lyons.

THE MATHEMATICAL ASSOCIATION.

THE annual meeting of the Mathematical Association was held in the London Day Training College, Southampton Row, on January 7 and 8, under the presidency of Prof. E. T. Whittaker. At the advanced section on the evening of January 7 the president gave a lecture on "A Survey of the Numerical Methods of Solving Equations." He described in some detail "iterative processes" for approximating to the roots and graphical methods of circumscribing the regions on the Argand plane, in which the various roots lay. The Lobachefsky-Graeffe method of approximating to the roots of equations and power series was described in considerable detail. In the animated discussion to which this lecture gave rise it was clearly seen that a wider knowledge of practical computative processes is a desideratum in all branches of mathematical work, which has been practically neglected hitherto in the schools and universities. It was also felt that such practical numerical work was the best possible introduction to the formal study of function theory, many of the ideas underlying which are usually presented in an entirely abstract way, whereas they present themselves naturally and of necessity in less general forms in the science of computation.

Next day, at the general section, Mr. C. Godfrey, of the Royal Naval College, Osborne, surveyed the whole question of the modern teaching of geometry in schools. He strongly favoured a preliminary course of practical instrumental work, to be followed by a more formal course in which "logic" is not too prominent. He advocated the entire postponement of a really rigorous course of abstract geometry until the post-school stage. Prof. T. P. Nunn strongly supported the general tenor of Mr. Godfrey's views, and urged the earlier teaching of "ratio and proportion" as a practical instrument for solving many problems, such as map-drawing, villa construction, etc.

Prof. E. H. Neville, of University College, Reading, next read a paper on "Convention and Duplexity in Elementary Mathematics," in which he protested against the usual "positive-sign" convention with regard to vectors. Miss H. M. Cook dealt with "The Place of Common Logarithms in Mathematical Training," and Prof. W. P. Milne strongly urged