material might be formed at such a temperature if some helium were present.

But of course the heat used up in forming these substances would cool the rest of the mass: any energy gained in radio-active form would be lost in the form of heat. It could never avail to explain a solar constant such as has been measured for longer than Kelvin's 20 million years. In other words, radioactive substances produced would act only as accumulators of energy, not as primary batteries.

To recapitulate: As Kelvin showed, gravitational energy can only account for 18·3 million years of sunshine at the present rate. Invoking radio-activity as a source of energy implies the assumption that unknown radio-active materials liberating considerably more energy than uranium were created by some unknown agency within a measurable period of time, and that these are now breaking up. This assumption is not necessary to account for the existence of uranium, as it is quite conceivable that a certain amount of radio-active matter might be produced afresh during every stellar collision. The energy of substances formed in this way would not be available to explain a greater amount of energy on the sun as their energy is abstracted from the gravitational energy, and has already been taken into account.

F. A. LINDEMANN.

Sidholme, Sidmouth, April 5.

Harmonic Analysis.

IN a paper which I read to the Physical Society last January (see NATURE, February 11, p. 662) I suggested that the best way of analysing a wave, the graph of which was given, was to apply the rules for the mechanical quadrature of integrals which are given in treatises in the calculus of finite differences. I am convinced that these methods when applied intelligently are much simpler and ever so much more accurate than most, if not all, of the methods in everyday use.

In the paper referred to above I applied a well-known method of mechanical quadrature (Weddle's rule) to the case of a semicircular alternating wave, the equation to the positive half of which is $y=\sqrt{x-x^2}$. I chose this wave because I found that the evaluation of the Fourier integrals for it by analysis was laborious. Prof. A. E. Kennelly, of Harvard University, has kindly written to me to point out that the equation to the curve can be readily put in the form—

$$y=J_1(\pi/2)\sin\pi x-(1/3)J_1(3\pi/2)\sin 3\pi x+(1/5)J_1(5\pi/2)\sin 5\pi x-\ .\ .\ .$$

where $J_1(x)$ is the Bessel's function of the first order. Hence from tables of these functions we get:—

$$y=0{\cdot}567\sin\pi x+0{\cdot}0939\sin 3\pi x.$$
$$+0{\cdot}0422\sin 5\pi x+0{\cdot}0252\sin 7\pi x.$$
$$+0{\cdot}0171\sin 9\pi x+\ .\ .\ .$$

Very close approximations to these numbers can be obtained very simply by Weddle's rule. For example, if b_1 denote the amplitude of the first harmonic, we have:

$$10b_1=5y_{1/6}+\sqrt{3}y_{1/3}+6y_{1/2},$$

where $y_n=\sqrt{n-n^2}$, and hence $b_1=0{\cdot}568$.

To get an accuracy of the same order for the third, fifth, seventh, and ninth harmonics we must calculate or measure the lengths of 8, 13, 18, and 23 ordinates respectively. Doing this, we find that $b_3=0{\cdot}0942$, $b_5=0{\cdot}0423$, and that b_7 and b_9 are given correctly. It will be seen that from the practical point of view the simplicity and accuracy of the method in this case leave little to be desired. It has the great advantage that the amplitude of each harmonic can be computed independently of the others.

When the wave passes smoothly through the extremities of the ordinates we measure, we can apply the rule with confidence. Jagged or very distorted waves must be treated more carefully. For example, if we apply the rule to a rectangular alternating wave of height unity we find from the formula given above that $10b_1=11+\sqrt{3}$, and so $b_1=1{\cdot}27321$ approx. The true value is $4/\pi$, *i.e.*, 1·27324 . . ., and hence the error is less than 1 in 40,000. For a triangular alternating wave of height unity, however, if we apply the rule intelligently we get $b_1=0{\cdot}88$ . . . instead of 0·81057 . . . The error in this case arises from applying Weddle's rule through a point of discontinuity. If we apply it over one-quarter of the wave, it being necessary to measure six ordinates instead of three, we find that $b_1=0{\cdot}81056$ . . .

ALEXANDER RUSSELL.

Faraday House, Southampton Row, W.C., April 12.

A Mistaken Butterfly.

REFERRING to Prof. Barnard's letter so titled in NATURE of April 15, which describes the apparent mistake of a butterfly in visiting a peacock's feather as if expecting to "extract food," I think it probable that there are no animals that do not make mistakes at times. I observed an analogous mistake made by a species of Pieridæ—*Appias nero*—in Sumatra, as I have recorded in "A Naturalist's Wanderings," p. 130:—"In the open paths I netted scarlet Pieridæ . . . often flying in flocks of over a score, exactly matching in colour the fallen [withered] leaves, which it was amusing to observe how often they mistook for one of their own fellows at rest, and to watch the futile attentions of an amorous male towards such a leaf moving in the wind."

HENRY O. FORBES.

Redcliffe, Beaconsfield, Bucks, April 17.

The "Green Ray" at Sunset.

PROF. A. W. PORTER, in NATURE of February 18 (vol. xciv., p. 672), seems to think that the "green ray" is more of a subjective phenomenon than anything else, or at least often is so; but the fact that it is seen at sunrise also shows that in this case at least it is not a result of complementary colours. Besides, if it were a subjective phenomenon, one would expect to see it on every occasion when the sun set behind a clear horizon, whereas the sight is somewhat rarer. I once saw a lovely blue flash, and I read a description recently of a sunset in Palestine where the writer speaks of the sun vanishing like a blue spark. If you hold a lens almost edgeways on between your eye and a light and move it until it is quite edgeways on a few discs of light will be seen, and at last these vanish in a green or blue flash, the effect of dispersion.

35 Roeland Street, Cape Town, March 17.