STAR CLUSTERS.

OF all the telescopic objects in the sky none are more beautiful or more fascinating than the condensed, globular star clusters. Their bewildering complexity renders them unsuitable for direct study at the telescope, but photography has now brought them within the range of systematic investigation. The technical problem which they present is by no means easy, and demands high resolving power for success. The fine examples reproduced herewith, M3 and M13, have been very kindly sent from the Mount Wilson Solar Observatory, and illustrate admirably the work of the famous 5 ft. mirror constructed by Mr. G. W. Ritchey.

Considerable attention was given to the star clusters by Sir John Herschel, whose attempts to depict them by hand met naturally with small success. Certain curious irregularities which he believed to exist in the distribution of the stars may be attributed to a purely subjective origin, or they may be accounted for by the absorptive influence of external dark nebulous masses. No great importance is now attached to them, and in the main the stars may be considered as distributed with radial symmetry. But one curious feature noticed by Sir John Herschel has been confirmed by later study. The stars in a cluster tend to divide into two classes of magnitude, a brighter and a fainter, separated by a distinct interval. Can this be a visible division of stars presumably at the same distance and of nearly equal age into the two classes of giant and dwarf stars inferred by Hertzsprung and H. N. Russell?

About twenty years ago Prof. S. I. Bailey, at that time at Arequipa, devoted considerable study to photographs of the chief globular clusters. His work proceeded on two lines. On one hand he made systematic counts of the stars recorded, thus laying the foundation for statistical investigations of their arrangement in space. And on the other he investigated the magnitudes of the stars, and was thus led to the remarkable discovery that several clusters contain a high proportion of variable stars, a ratio of 1 in 7 in the extreme case of M3. His detailed results for the clusters ω Centauri and M3 have been published in two beautiful memoirs. The type of variation is of a distinct character, though a few isolated examples have been found elsewhere in the sky, with a period of about twelve hours and a rapid rise to maximum. In the case of M3 the variation is singularly true to one type, the range between maximum and minimum being two photographic magnitudes. Some clusters, notably M13, are almost entirely devoid of such variables; where they do occur they are apparently confined to the stars of the brighter order of magnitude.

The question of the distribution of stars in clusters was discussed by Prof. E. C. Pickering. Using counts on the clusters ω Centauri, 47 Tucanæ and M13 (Herculis), he formed the important conclusions: (1) that the law of distribution is essentially the same for different clusters, (2) that the bright stars and the faint stars of a cluster obey the same law. He represented graphically the curve of apparent (projected) density for different distances from the centre, and attempted without success to reproduce it by assuming laws of the form $1-r^2$ and $(1-r)^n$ for the density in space. The latter form was also tested by Mr. W. E. Plummer with much the same result on an extensive series of measures of the stars in M13.

FIG. 1.—M3 Canes Venatici. Exposure 4h.

The next important contribution to the subject is due to H. v. Zeipel, who measured the positions of the stars in M3 (Can. Ven.). By adapting the solution of a certain integral equation studied by Abel he showed how the law of distribution in space may be deduced numerically from the observed distribution as it is seen in projection. Later he compared the law of density in space arrived at in this way with that which obtains in a gravitating spherical mass of gas in isothermal equilibrium. The result represents the density of the cluster satisfactorily near the centre, but in the outer regions the cluster is less dense than the theory requires.

The physical conception thus introduced suggested other possibilities. A sphere of gas in adiabatic, instead of isothermal, equilibrium might be chosen as the standard of comparison. A series of states exists, depending on the constant ratio γ of the specific heats of the gas, which have been extensively studied by Lord Kelvin and others. Emden's "Gaskugeln" is a work dealing exhaustively with the subject. In general, the law of density cannot be expressed in finite terms. But there are exceptional cases in which the differential equation possesses a very simple solution. One of these, discovered by Schuster, corresponds to the value $\gamma = 1{\cdot}2$. Here the law expressing the density at the distance r from the centre takes the form:

$$3a^2N/4\pi(a^2+r^2)^{5/2},$$

where N is the total mass or number of stars. This is finite, although the distribution extends to infinity. If a finite boundary be expected it is impossible to fix one by the counts, and attempts to do so have been proved illusory by the occurrence of characteristic variable stars beyond the supposed limit. However this may be, a comparison of the law with Bailey's counts of the ω Centauri cluster showed immediately an agreement within the limits within which radial symmetry is observed. I next compared the law with Pickering's curve of the projected densities, based on the clusters ω Centauri, M13 and 47 Tucanæ (bright and faint stars treated separately). The accordance was again excellent, and left little doubt that the law represented much more than a mere formula of interpolation. When, however, v. Zeipel's counts of M3 were examined, the outer region was found to conform with the law, while the inner revealed a higher density than was to be expected. As v. Zeipel had, on the other hand, succeeded in representing the central distribution by the isothermal law, it was suggested that the true standard of comparison was a central isothermal core surrounded by an adiabatic envelope, a composite state of equilibrium actually contemplated by writers on the thermodynamics of the subject. Afterwards, by the use of similar methods, Prof. Strömgren proved that M5 (Serpentis) possesses a structure which, whatever the cause, is identical with that of M3. v. Zeipel remarked that the excessive central condensation was more marked among the bright than among the faint stars.

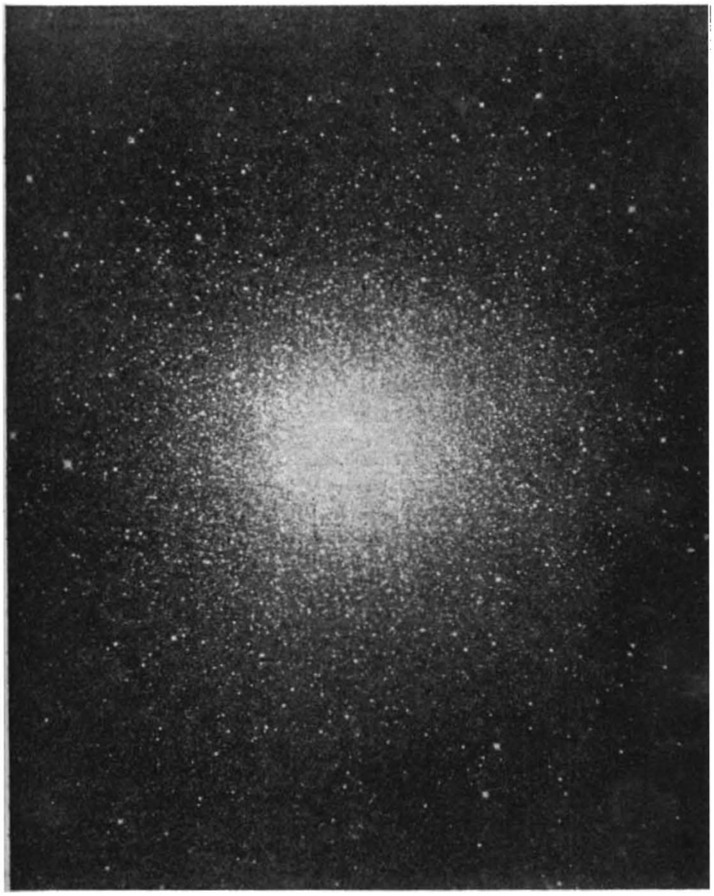

FIG. 2.—M 13 Herculis. Exposure 11h. (3 nights).

The problem has again been discussed by v. Zeipel in an elaborate memoir, using in this instance counts of the stars in M2 (Aquarii), M3, M13 and M15 (Pegasi). He first finds solutions corresponding to these values of γ:

(M2) 1·200, (M3) 1·156, (M13) 1·183, (M15) 1·179

Thus M2 conforms with the same simple law, which I had found to hold so perfectly for ω Centauri. On the other hand, M3 is again seen to depart from it, and even with the new value of γ the representation is far from good. The law of density here contemplated is a solution of the equation:

$$\frac{d^2(r\rho^{\gamma-1})}{dr^2}+r\rho=0,$$

and satisfies a physical condition in being regular at the centre. The general solution, however, possesses a singularity at this point, and contains an additional arbitrary constant. Thus the particular law given above is only a special case of the general solution for $\gamma = 1{\cdot}2$, which, as v. Zeipel shows, can be expressed in elliptic functions. Accordingly, he abandons the central condition, and introduces the additional constant which is to be determined, together with γ, for each case. With this modification of the theory the values of γ became:

(M2) 1·194, (M3) 1·198, (M13) 1·203, (M15) 1·197, so that within the limits of uncertainty in every case the distribution of the stars is consistent with a solution of the above differential equation when γ is assigned the value 1·2.

The analogy between the distribution of stars in a condensed cluster and the density in a spherical mass of gas of a particular type in adiabatic equilibrium thus seems to be fairly established. Even if it be supposed that the cluster is the outcome of an original nebula the question still remains why the distribution of matter should persist long after its condition has completely changed, or why the arrangement should resemble what might be expected of certain vapours (*e.g.*, chloroform). The answer given by v. Zeipel on the basis of a strict mathematical analysis is that this is in conformity with a kinetic theory which applies to an aggregate containing a high proportion of Keplerian binaries. This may be a bold application of the law of large numbers, but it is certainly an interesting conception. Since there is every reason to believe that all short period variables are binary systems the observed occurrence of these in clusters lends support to the view, though they can only represent the exceptionally close systems. The investigations here described refer exclusively to the highly condensed clusters. But there exist also clusters showing states of concentration in varying degree until probably all visible trace of organic connection is lost. In Strömgren's view the whole series represents an order of evolution by which the dense clusters grow out of more scattered forms. Whether the results will throw light on the wider problems of the structure of the sidereal universe seems doubtful in view of certain conclusions drawn by Poincaré, Jeans and Eddington as to the relevance of the kinetic theory. But taken by themselves they present questions of the highest interest which are likely to repay further study.

H. C. PLUMMER.

ON COLOUR SENSITISED PLATES.

I.—IN GENERAL AND ORTHOCHROMATIC PLATES.

IT used to be customary to draw three curves above a diagrammatic spectrum, heat, luminosity, and actinism curves, the last representing the power of light to produce or facilitate chemical change independently of the temperature change. This custom survives to a certain extent, though only one of the curves, namely, the heat curve, is definite. The luminosity curve depends upon the human eye, and eyes vary, sometimes even in the same individual, with regard to their sensitiveness to light and colour. Still, it is possible to draw practically useful luminosity curves in a general sense, and by taking an average human eye, in perhaps almost an absolute sense.

But the "actinism" curve is essentially different, for here we may be concerned, not with a single organ and its possible variations or degrees of perfection, but with every substance that exists on the face of the earth or that can be prepared by artificial means. And if we limit our considerations to the very few substances that are practically utilised in photography, we find that "actinism" extends from well into the infra-red down to the Röntgen rays, which are far below what is generally known as the ultra-violet. "Actinism" extends over a range of eleven or twelve octaves for practical photographic purposes, while luminosity extends over scarcely one octave, and for practical purposes even less than this, and yet some people speak of the photographic plate as colour-blind!

The whole of this eleven or twelve octaves has not yet been dealt with photographically, because in the extreme ultra-violet (the "Schumann region") at wave-lengths a little less than 200 $\mu\mu$, the absorbing power of air and gelatine prevents the passage of radiations through them. But this appears to be due to absorption bands, as radiations of still shorter wave-length (Röntgen rays) pass freely through these media. By getting rid as far as possible of air and gelatine, the photography of the ordinary spectrum has been extended down to wave-length 100$\mu\mu$, or even less. There are other difficulties than the air and gelatine to contend with in investigations of this region, but with these we are not immediately concerned.

Although it is necessary sometimes to bear in mind the enormous range of sensitiveness of photographic materials, even from a purely practical point of view, if we exclude the Röntgen region, and regard only those circumstances that concern the photography of objects, whether terrestrial or celestial, and whether by daylight or artificial light, we have to consider only about two octaves of radiations, or rather more if the far infra-red is taken into account. This range may be still further curtailed when daylight or glass apparatus is used, on account of the absorptive power of glass and the atmosphere, and what remains may often be sufficiently described by indicating five regions, namely, ultra-violet, blue, green, red, and infra-red. The "blue" will include the indigo and violet, and the "red" will include the orange, and the yellow is negligible as in a good spectrum it is represented by little more than the sodium D lines.

In order to photograph coloured objects so that their luminosities shall be correctly represented in the print, we want to get the curve that represents the action of the spectrum on the plate to coincide with the luminosity curve of the spectrum, and then we want a printing method that will preserve these tone values. The alternative of getting equal and opposite errors in the negative and the print so that the one shall correct the other, may have a degree of possibility about it. The fact to be emphasised is that the getting of a correct negative is not the whole business. Indeed, the getting of the two curves to correspond is not the whole business so far as the negative is concerned, for they may correspond at one exposure of the plate to the spectrum and not at another, because the steepness of the gradation of the deposits produced on the plate by equivalent ranges of exposures to the various parts of the spectrum is not the same. These difficulties are mentioned to show that, from a practical point of view, "ortho-