LETTERS TO THE EDITOR.

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Reflection of Light at the Confines of a Diffusing Medium.

I SUPPOSE that everyone is familiar with the beautifully graded illumination of a paraffin candle, extending downwards from the flame to a distance of several inches. The thing is seen at its best when there is but one candle in an otherwise dark room, and when the eye is protected from the direct light of the flame. And it must often be noticed when a candle is broken across, so that the two portions are held together merely by the wick, that the part below the fracture is much darker than it would otherwise be, and the part above brighter, the contrast between the two being very marked. This effect is naturally attributed to reflection, but it does not at first appear that the cause is adequate, seeing that at perpendicular incidence the reflection at the common surface of wax and air is only about 4 per cent.

A little consideration shows that the efficacy of the reflection depends upon the incidence not being limited to the neighbourhood of the perpendicular. In consequence of diffusion [1] the propagation of light within the wax is not specially along the length of the candle, but somewhat approximately equal in *all* directions. Accordingly at a fracture there is a good deal of "total reflection." The general attenuation downwards is doubtless partly due to defect of transparency, but also, and perhaps more, to the lateral escape of light at the surface of the candle, thereby rendered visible. By hindering this escape the brightly illuminated length may be much increased.

The experiment may be tried by enclosing the candle in a reflecting tubular envelope. I used a square tube composed of four rectangular pieces of mirror glass, 1 in. wide, and 4 or 5 in. long, held together by strips of pasted paper. The tube should be lowered over the candle until the whole of the flame projects, when it will be apparent that the illumination of the candle extends decidedly lower down than before.

In imagination we may get quit of the lateral loss by supposing the diameter of the candle to be increased without limit, the source of light being at the same time extended over the whole of the horizontal plane.

To come to a definite question, we may ask what is the proportion of light reflected when it is incident equally in all directions upon a surface of transition, such as is constituted by the candle fracture. The answer depends upon a suitable integration of Fresnel's expression for the reflection of light of the two polarisations, viz. :—

$$S^2=\frac{\sin^2(\theta-\theta')}{\sin^2(\theta+\theta')},\quad T^2=\frac{\tan^2(\theta-\theta')}{\tan^2(\theta+\theta')},\quad . \quad . \quad (1)$$

where θ, θ' are the angles of incidence and refraction. We may take first the case where $\theta>\theta'$, that is, when the transition is from the less to the more refractive medium.

The element of solid angle is $2\pi \sin\theta\, d\theta$, and the area of cross-section corresponding to unit area of the refracting surface is $\cos\theta$; so that we have to consider

$$2\int_0^{\frac{1}{2}\pi} \sin\theta\cos\theta\,(S^2 \text{ or } T^2)\,d\theta,\quad . \quad . \quad . \quad (2)$$

[1] To what is the diffusion due? Actual cavities seem improbable. Is it chemical heterogeneity, or merely varying orientation of chemically homogeneous material operative in virtue of double refraction?

the multiplier being so chosen as to make the integral equal to unity when S^2 or T^2 have that value throughout. The integral could be evaluated analytically, at any rate in the case of S^2, but the result would scarcely repay the trouble. An estimate by quadratures in a particular case will suffice for our purposes, and to this we shall presently return.

In (2) θ varies from 0 to $\frac{1}{2}\pi$ and θ' is always real. If we supppose the passage to be in the other direction, viz. from the more to the less refractive medium, S^2 and T^2 being symmetrical in θ and θ', remain as before, and we have to integrate

$$2 \sin\theta' \cos\theta' \,(S^2 \text{ or } T^2)\,d\theta'.$$

The integral divides itself into two parts, the first form 0 to a, where a is the critical angle corresponding to $\theta=\frac{1}{2}\pi$. In this S^2, T^2 have the values given in (1). The second part of the range from $\theta'=a$ to $\theta'=\frac{1}{2}\pi$ involves "total reflection," so that S^2 and T^2 must be taken equal to unity. Thus altogether we have

$$2\int_0^a \sin\theta'\cos\theta'\,(S^2 \text{ or } T^2)\,d\theta'+2\int_a^{\frac{1}{2}\pi}\sin\theta'\cos\theta' d\theta',\quad (3)$$

in which $\sin a=1/\mu$, μ (greater than unity) being the refractive index. In (3)

$$2\sin\theta'\cos\theta' d\theta'=d\sin^2\theta'=\mu^{-2}d\sin^2\theta,$$

and thus—

$$(3)=\mu^{-2}\times(2)+1-\mu^{-2}$$
$$=\frac{1}{\mu^2}\left\{\mu^2-1+\int_0^{\frac{1}{2}\pi}\sin 2\theta\,(S^2 \text{ or } T^2)\,d\theta\right\},\quad (4)$$

expressing the proportion of the uniformly diffused incident light reflected in this case.

Much the more important part is the light totally reflected. If $\mu=1{\cdot}5$, this amounts to 5/9, or 0·5556.

With the same value μ, I find by Weddle's rule—

$$\int_0^{\frac{1}{2}\pi}\sin 2\theta\,.\,S^2 d\theta=0{\cdot}1460,\qquad \int_0^{\frac{1}{2}\pi}\sin 2\theta\,.\,T^2 d\theta=0{\cdot}0339.$$

Thus for light vibrating perpendicularly to the plane of incidence—

$$(4)=0{\cdot}5556+0{\cdot}0649=0{\cdot}6205;$$

while for light vibrating in the plane of incidence—

$$(4)=0{\cdot}5556+0{\cdot}0151=0{\cdot}5707.$$

The increased reflection due to the diffusion of the light is thus abundantly explained, by far the greater part being due to the total reflection which ensues when the incidence in the denser medium is somewhat oblique.

RAYLEIGH.

The Pressure of Radiation.

THE theory of radiation at present accepted is based on Maxwell's result that the pressure of any component frequency is one-third of its energy density, which appears to result from an assumption analogous to Boyle's law, according to which the excess pressure due to vibration, in the case of a gas, would be one-third of the energy density of the vibration.

Lord Rayleigh (*Phil. Mag.*, 1905) has shown that this cannot be true in the case of a gas, since the vibrations are adiabatic, and Boyle's law does not hold. For a monatomic gas, where the reasoning based on the kinetic theory is fairly certain, he deduces that the excess pressure should be two-thirds of the energy density.

In a recent note on radiation and specific heat (*Phil. Mag.*, October, 1913) I gave an outline of a new theory, showing good agreement with experiment, from which I deduced the result that "the total pressure of full radiation should be one-third of the intrinsic energy density, but this could not be true for

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