the time of experiment in the uranium-radium series. It is easily shown, provided all the periods are long, that the initial rate of production must be proportional to the same power of the time as the number of substances (including uranium itself) in the series before radium. These experiments, therefore, indicate that there is only one intermediate substance in the uranium-radium series with a long period of life. Assuming what is probable, but not yet known, that the present law and rate of production will be continued in the future, it is possible to fix the period of average life of the intermediate substance from the existing data, with a margin of uncertainty probably not greater than 20 per cent. Rutherford has shown that the initial production of radium in grams from a kilogram of uranium is equal to $6 \times 10^{-8} \lambda T^2$, where $1/\lambda$ is the period of average life of the intermediate body in years, and T is the time in years. This gives for the average life of the intermediate body the period of 10,000 *years*. This is about four times that of radium itself, and there should exist in uranium minerals about 1·36 milligrams of the substance per kilogram of uranium. The initial rate of production over the first two years appears less than that calculated, as though another intermediate substance of period of the order of two years existed in the series; but greater refinements will be necessary before this can be definitely proved by experiments of this character.

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Wave Motion and Bessel's Functions.

THE property, enunciated by Dr. Johnstone Stoney, according to which any wave motion can be regarded as built up of a combination of plane waves, may be used with advantage for a verification of the formulæ for the solutions of Bessel's equation in the form of definite integrals.

Consider, for example, the hydrodynamical problem of circular waves about the axis of z in a liquid of uniform depth extending from $z=0$ up to the free surface $z=h$. Imagine the wave motion to be built up by the superposition of a continuously infinite number of plane waves, symmetrically distributed about the axis of z. By taking α to denote the angle which the normal to any wave front makes with the radius vector to any point, and by integrating the expression for the velocity potential of the corresponding train of plane waves with respect to α, we get the expression

$$\phi = A\int_{-\pi}^{\pi} \cos m\{r\cos\alpha - vt + \epsilon\} \cosh mz \, d\alpha,$$

where

$$v^2 = \frac{g}{m} \tanh mh.$$

The above expression for ϕ being a solution of Laplace's equation, it follows that

$$\int_{-\pi}^{\pi} \cos m\,(r\cos\alpha)\,d\alpha \text{ and } \int_{-\pi}^{\pi} \sin m\,(r\cos\alpha)\,d\alpha$$

are solutions of the corresponding Bessel's equation in r.

Next, taking an unsymmetrical distribution of plane waves, and confining attention to the particular case in which the phase relative to the origin is independent of the direction, the amplitude between the directions α and $\alpha + d\alpha$ being $F(\alpha)d\alpha$, we find for the potential at the point (r, θ, z) the expression

$$\phi = \int F(\alpha) \cos m\,\{r\cos(\alpha-\theta) - vt + \epsilon\} \cosh mz \, d\alpha,$$

the integral being taken between limits for α differing by 2π. By writing $\alpha - \theta = \omega$, and suitably choosing the limits, we find

$$\phi = \int_{-\pi}^{\pi} F(\theta+\omega) \cos m\,\{r\cos\omega - vt + \epsilon\} \cosh mz \, d\omega,$$

and taking the particular cases of $F(\alpha) = \cos$ or $\sin n\alpha$, we obtain the solutions of Laplace's equation

$$\cosh mz \cos \text{ or } \sin n\theta \int_{-\pi}^{\pi} \cos \text{ or } \sin n\omega \cos \text{ or } \sin m\,(r\cos\omega)\,d\omega$$

leading to solutions of Bessel's equation of order n, namely,

$$\int_{-\pi}^{\pi} \cos \text{ or } \sin n\omega \cos \text{ or } \sin m\,(r\cos\omega)\,d\omega.$$

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Dew-Ponds.

LIKE "E. A. M." in NATURE of April 22, I have always been extremely sceptical about Mr. Hubbard's theory of dew-ponds since it first appeared in "Neolithic Dew-ponds and Cattle-ways." My own experience of lakes and ponds is that they lose their heat slowly, and that, after radiation has set in at night, they indicate a much higher temperature than the ground adjoining or the air above.

It has been a matter of frequent observation on Coniston Lake in summer that, after a night of heavy dew, the bottoms of the boats inside were perfectly dry, whilst the gunwale was covered with moisture, showing that the portion of the boat in contact with the water had been raised to a temperature above the dew-point. Prof. Miall and myself a few years ago made a special expedition to the Berkshire downs, in the neighbourhood of Wantage, to determine the temperature of the dew-ponds, and we found precisely the same thing, that is to say, the water at night was warmer than the air. It is impossible, therefore, that dew could deposit on the ponds under these conditions.

Moreover, as "E. A. M." points out, it is inconceivable that the clay or straw in a full dew-pond can have much connection with the temperature of the water. My own conviction is that the straw is merely used to bind the clay, and the bed of clay above the chalk serves no other purpose than to make the pond bottom water-tight. No satisfactory explanation of dew-ponds has yet been propounded, and, as your correspondent says, "there is room for more experiment." I have seen no reference to what may, I think, constitute one important factor in the replenishment of dew-ponds. Aitken has shown that the greater portion of the moisture deposited as dew is derived from the ground and not from the air, and in this connection it should be remembered that the chalk, on which the ponds are usually constructed, absorbs water like a sponge. Consequently, any lowering of temperature may cause a heavier dew or mist formation than on less absorbent material. Seeing that many of the ponds lie quite exposed on the very summit of the downs, drainage of dew cannot feed them, and it seems probable that mist may in some cases play a more important *rôle* than dew.

J. B. COHEN.

The Imperial Side of the Fuel Question.

THE article in NATURE of May 6, and Sir W. Ramsay's comment upon it, direct attention to a most important economic question. It has often crossed my mind that by a simple legislative enactment a marked saving might be effected in our factory consumption of coal. If Parliament would enact that after a given year, say 1920, a considerable penalty should be payable by the owner of any factory where the consumption of fuel coal exceeded 1½ lb. per hour per indicated horse-power, it is probable that almost all factories would by that date be improved up to that level of efficiency.

It is probable that the average efficiency of steam plants is only about 3 lb. per indicated horse-power hour, and your article shows the factory consumption to be about sixty-one million tons per annum. On those figures, halving the consumption on the above lines would save about thirty million tons a year. The modernisation of plant involved would pay for itself (from the factory owner's point of view) in a very few years, and so would be a remunerative investment, so much so that financing the change should be within reach of even the weaker firms.

The thirty million tons I suggest might be saved is more than 11 per cent. of the production (figures of 1907), so that the saving is well worth the attention of all who are concerned to conserve our coal, and I trust that the idea may be pressed forward in influential quarters.

Manchester, May 10. ARTHUR MCDOUGALL.