LETTERS TO THE EDITOR.

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Classification of Quartic Curves.

THE best method of classifying curves is to commence with one which is founded on properties which are unaltered by projection. We thus obtain ten principal species of quartic curves, viz. *anautotomic*, *uninodal*, *unicuspidal*, *binodal*, *nodocuspidal*, *bicuspidal*, *trinodal*, *binodocuspidal*, *nodobicuspidal* and *tricuspidal*; but each of these species admits of a variety of subsidiary divisions, owing to the fact that all curves of a higher degree than the third may possess compound singularities.

Anautotomic, unicuspidal, bicuspidal and tricuspidal quartics admit of a subsidiary division depending on the number of points of undulation they possess; and it must be borne in mind that, although it is convenient to use the term point of undulation, it is the tangent at this point and not the point itself which is the actual singularity.

Uninodal quartics admit of *three* primary subdivisions, according as the double point is an ordinary node, a flecnode or a biflecnode.

Binodal quartics admit of *seven* primary subdivisions, six of which depend on the character of the node, whilst the seventh arises from the fact that the two nodes may unite into a tacnode.

Nodocuspidal quartics admit of only *four* primary subdivisions, three of which depend on the character of the node, whilst the fourth arises from the fact that the node and cusp may unite into a rhamphoid cusp.

Trinodal quartics admit of *ten* primary subdivisions, and in order to particularise them, we shall denote the different singularities which involve a double point by their initial letters, except that *tp* and *tc* will be used to denote a triple point and a tacnode cusp respectively; so that the nomenclature *n*, *n*, *n* and *n*, *n*, *f* will indicate that the quartic has three nodes or two nodes and a flecnode respectively. We shall then have the following ten species:—(1) *n*, *n*, *n*; (2) *n*, *n*, *f*; (3) *n*, *n*, *b*; (4) *n*, *f*, *f*; (5) *b*, *b*, *b*; (6) *t*, *n*; (7) *t*, *f*; (8) *t*, *b*; (9) *o*; (10) *tp* of the first kind.

Binodocuspidal quartics admit of *eight* primary subdivisions, which are as follows:—(1) *c*, *n*, *n*; (2) *c*, *n*, *f*; (3) *c*, *f*, *f*; (4) *t*, *c*; (5) *r*, *n*; (6) *r*, *f*; (7) *tc*; (8) *tp* of the second kind.

Nodobicuspidal quartics admit of *three* primary subdivisions, which are:—(1) *c*, *c*, *n*; (2) *c*, *r*; (3) *tp* of the third kind

Whenever any of these primary species represents a curve which has two or more points of inflection, a further subdivision may usually be made which depends upon the number of points of undulation it can possess. Thus the species *n*, *n*, *n* may possess two, one or no points of undulation; whilst the species *c*, *c*, *n* may possess one or no such points.

A fourth subdivision may sometimes be made which depends upon whether the quartic is capable of being projected into a curve which is symmetrical or hemisymmetrical with respect to a pair of rectangular axes. In some cases, the possibility of the projection involves the existence of compound singularities, and thus the curve belongs to one of the species already considered but in other cases, the necessary conditions do not affect the singularities. Thus all trinodal quartics which are capable of projection into symmetrical curves must belong to the species *n*, *n*, *b*; *b*, *b*, *b*; or *t*, *b*, in which three respective cases the quartic can be projected into the inverse of an ellipse or hyperbola with respect to its centre, the lemniscate of Bernoulli or the lemniscate of Gerono. On the other hand, the possibility of projecting any quartic with three double points into a hemisymmetrical curve depends upon whether it can be projected into the inverse of a conic with respect to a point in its axis. The conditions for this do not necessarily involve compound singularities, since these will only exist for special positions of the centre of inversion.

There is no necessity to adopt a classification founded on the nature of the branches at infinity, since all the results can be obtained by projection. Thus, if a straight line cutting in four real points any quartic, which is unipartite and perigraphic, be projected to infinity, the projection will be quadripartite and will have four real asymptotes; and by taking special positions for the line to be projected, a variety of special results can be obtained. By projecting a triple point or a pair of crunodes to infinity, it is at once seen that a quartic can have three parallel or two pairs of parallel asymptotes. Also, if the polar cubic of a point *o* breaks up in to a conic and a line cutting the quartic in four ordinary points and the line be projected to infinity, the projection will have four asymptotes meeting in a point.

A quartic having three acnodes is the limiting form of an anautotomic quartic in which the acnodes are replaced by three perigraphic curves; and if a line cutting the fourth portion in four real points be projected to infinity, the projection will be septipartite. From this it appears that the partivity of a curve of the nth degree cannot be less than $n+\frac{1}{2}(n-1)(n-2)$.

A. B. BASSET.

Fledborough Hall, Holyport, Berks, November 14.

The Conservation of Mass.

APROPOS of the recent discussion on the conservation of mass at the Belfast meeting of the British Association, the following calculation may be of interest; it relates to the loss of weight undergone by a body when raised vertically.

If g is the acceleration of gravity at a specified point on the surface of the Earth, m the mass of a body of weight w, then

$$w=mg.$$

Now let the centre of gravity of the body be raised through a vertical distance d; g will be changed into

$$g'=\left(\frac{R}{R+d}\right)^2 g,$$

R being the radius of the Earth (supposed spherical), and the corresponding weight of the body will be

$$w'=mg'$$

on the supposition of the conservation of mass.

The loss of weight is thus

$$\delta=w-w'=w\left\{1-\frac{R^2}{(R+d)^2}\right\}$$

$$=w\left\{1-\left(1+\frac{d}{R}\right)^{-2}\right\}=\frac{2dw}{R},$$

neglecting second and higher powers of $\frac{d}{R}$.

As a particular example, take $w=1$ kilogm., $d=10$ cm. and R approx. $=6357\times 10^5$ cm.

Then

$$\delta=0{\cdot}00003 \text{ gm}.$$

[The term involving $\left(\frac{d}{R}\right)^2$ would have the first significant figure in the fifteenth place, and therefore we were justified in this case in neglecting it.]

This small difference is, I believe, of the same order as those which Prof. Landolt found; but the ratio of the difference to the whole weight (*i.e.* $2d:R$) must have been much greater in his experiments. Although Prof. Landolt's discrepancies may receive a perfectly different explanation, it is quite conceivable that a balance could be constructed which would detect such small differences. It is scarcely necessary to point out that, in the actual performance of the experiment, the scale-pan containing the counterpoising weights must be at the same height during the two weighings. D. M. Y. SOMMERVILLE.

St. Andrews, November 12.

A Simple Experiment in Diffraction.

M. G. FOUSSERAU describes, in the *Journal de Physique* for October, a simple apparatus for viewing diffraction and interference phenomena, a modified form of which I have experimented on with success. In the latter form, the source of light was obtained by placing a diaphragm on the stage of a microscope, on which sunlight was concentrated by means of the mirror and condenser, and the diffraction effects were produced by placing perforated pieces of tinfoil on the top of the microscope tube where the eye-piece is usually placed. On placing the eye close up to the tiny hole in the tinfoil, various diffraction patterns were seen. The difficulty of piercing a hole that is truly circular in tinfoil made it hard to obtain perfect rings, but the "failures" were often very interesting. A rectangular aperture