to any particular group of nerve-cells. Moreover, the anatomical evidence of such changes taking place is at present of the flimsiest character.

If these theories be true, what, it may be asked, is the agency that causes the dendrites to contract or the neuroglia cells to expand? Is there really a soul sitting aloof in the pineal gland, as Descartes held? When a man like Lord Brougham can at any moment shut himself away from the outer world and fall asleep, does his soul break the dendritic contacts between cell and cell; and when he awakes, does it make contacts and switch the impulses evoked by sense stimuli on to one or other tract of the axons, or axis cylinder processes, which form the association pathways? Such an hypothesis is no explanation: it simply puts back the whole question a step further, and leaves it wrapped in mystery. It cannot be fatigue that produces the hypothetical interruptions of the dendritic synapses and then induces sleep, for sleep can follow after fatigue of a very limited kind. A man may sleep equally well after a day spent in scientific research, as after one spent in mountain-climbing, or after another passed in idling by the sea-shore. He may spend a whole day engaged in mathematical calculation, or in painting a landscape. He fatigues—if we admit the localisation of function to definite parts of the brain—but one set of association tracts, but one group of cells, and yet, when he falls asleep, consciousness is not partially, but totally suspended.

We must admit that the withdrawal of stimuli, or their monotonous repetition, are factors which do undoubtedly stand out as primary causes of sleep. We may suppose, if we like, that consciousness depends upon a certain rate of vibration which takes place in the brain structure. This vibration is maintained by the stimuli of the present, which awaken memories of former stimuli, and are themselves at the same time modified by these. By each impulse streaming into the brain from the sense organs, we can imagine the structure of the cerebral cortex to be more or less permanently altered. The impulses of the present, as they sweep through the association pathways, arouse memories of the past; but in what way this is brought about is outside the range of explanation. Perhaps an impulse vibrating at a certain rate may arouse cells or fibrils tuned by past stimuli to respond to this particular rate of vibration. Thus may be evoked a chain of memories, while by an impulse of a different rate, quite another set of memories may be started. Tracts of association are probably formed in definite lines through the nervous system, as during the life of a child repeated waves of sense-impulses beat against and overcome resistances, and make smooth pathways here and there through the brain structure. Thus may be produced growth of axons in certain directions, and synapses of this cell with that. If the same stimulus be often repeated, the synapses between groups of cells may become permanent. A memory, a definite line of action which is manifested by a certain muscular response, may thus become structurally fixed. If the stimulus be not repeated, the synapses may be but temporary, and the memory fade as the group of cells is occupied by a new memory of some more potent sense stimulus. Many association tracts and synapses are laid down in the central nervous system when the child is born. These are the fruits of inheritance, and by their means, we may suppose, instinctive reflex actions are carried out.

So long as the present stimuli are controlled by past memories and are active in recalling them, so long does consciousness exist, and the higher will be the consciousness the greater the number and the more intense the character of the memories aroused. We may suppose that when all external stimuli are withdrawn, or the brain soothed by monotony of gentle repetition, and when the body is placed at rest, and the viscera are normal and give rise to no disturbing sensations, consciousness is then suspended, and natural sleep ensues. Either local fatigue of the muscles, or of the heart, or ennui, or exhaustion of some brain centre usually leads us to seek those conditions in which sleep comes. The whole organism may sleep for the sake of the part. To avoid sleeplessness, we seek monotony of stimulus either objective or subjective. In the latter case, we dwell on some monotonous memory picture, such as sheep passing one by one through a gap in the hedge. To obtain our object, we dismiss painful or exciting thoughts, keep the viscera in health, so that they may not force themselves upon our attention, and render the sense-organs quiet by seeking darkness, silence and warmth. L. H.

A PROPOSED REVOLUTION IN NAUTICAL ASTRONOMY.

DURING the last two years a movement has been set on foot, which seems likely to be attended by somewhat important results in the simplification of the formulæ of astronomical navigation for every-day use. Any one who has looked even cursorily into a text-book of navigation of the Raper type, can hardly fail to have been impressed by the multiplicity and variety of the precepts, and can easily understand how complicated the various rules must appear to the unlearned men, upon whom, for the most part, the daily routine of practical navigation at sea must devolve.

And the difficulty of comprehending and putting into practice the various rules, is undoubtedly increased by the fact that at one time or another all the trigonometrical functions of an angle are brought into play. Sines, cosines, tangents, cotangents, secants and cosecants, versed sines and half-versed sines, all make their appearance, adding to the bewilderment of the unskilled computer, and introducing the liability to take a required function from a wrong column as a very frequent source of error.

Nautical astronomy, for the most part, may be regarded as simply a practical application of the formulæ employed in the solution of spherical triangles, so that the object to be attained by those who would simplify the various problems, is to devise a system of formulæ in logarithmic shape, which, without materially adding to the amount of arithmetic employed, should introduce but one function of an angle throughout, such as the sine, the cosine, or the tangent. In the verbal precepts, into which, for the benefit of those possessing no knowledge of mathematics, the formulæ have to be translated, the simple word "logarithm" would then take the place of "log sine," "log cosine," &c., and a single table of a few thousand logarithms would do the work formerly effected by the aid of a large collection of different tables.

To M. E. Guyou, an officer of the French navy, belongs the credit of having first devised such a system. As far back as the year 1885 he published in a small pamphlet entitled "Tables de Poche," methods of finding hour angle and azimuth of a heavenly body by means of a single table of logarithms. During the next ten years he employed himself in further researches, and early in 1896 there appeared in connection with the "Annales Hydrographiques," published periodically by the Hydrographic Department of the French navy, a more exhaustive account of his methods, with a special arrangement of the required table, intended to enable his processes to be more easily and effectively carried out.[1]

The particular table employed by M. Guyou does not give logarithms for one of the ordinary functions of the

[1] "Les problèmes de Navigation et la Carte Marine. Types de calcul et tables complètes." Par M. le capitaine de frégate E. Guyou, Membre de l'Académie des Sciences. (Paris: Imprimerie Nationale, 1895.)

angle, but is a table which is made use of daily in the calculations which belong to Mercator sailing, and which is consequently to be found in every collection of nautical tables. It is known as the table of "meridional parts," or, as the French call it, "*latitudes croissantes.*" The meridional parts for a given latitude are defined by some writers as "the value in minutes of a great circle of the line on the Mercator's chart, into which the true difference of latitude is expanded."

For a given latitude l the meridional parts represent the sum of the series

$$\sec 0' + \sec 1' + \sec 2' + \sec 3' + \ldots + \sec (l^{\circ} - 1')$$

which is found by the integral calculus to be

$$r \log_e \tan \left(45^{\circ} + \frac{l}{2}\right)$$

or

$$\frac{10800}{\pi} \log_e \tan \left(45^{\circ} + \frac{l}{2}\right)$$

when r is expressed in minutes.

In the table of meridional parts we have then a series of logarithms to the base $e^{\frac{\pi}{10800}}$, which has been found to lend itself in a remarkable manner to the purpose which we have in view.

It should be mentioned here that M. Guyou's general method is to deduce his formulæ from a study of the properties of the curves of equal altitude on a Mercator's chart. To other writers, especially in Italy, where considerable attention has been bestowed upon the new formulæ, it has appeared more satisfactory, while accepting the expressions, to deduce them directly from fundamental trigonometrical formulæ.

Shortly before the issue of M. Guyou's second work there was published, in the numbers of the *Nautical Magazine* for November and December 1895, a system of formulæ, for the solution of all the ordinary problems of nautical astronomy, by the aid of this table of meridional parts alone, the general principle adopted being to break up the spherical triangle, or "triangle of position," as it is generally called in nautical astronomy, into two right-angled triangles, and thus obtain expressions which, containing three terms only, would be more manageable than the general formulæ involving four terms.

This treatment of the subject was based upon certain easily established lemmas, the most important of which may be thus stated. (The abbreviation MP will be adopted for meridional parts throughout.)

$$\mathrm{MP}(180^{\circ} - \theta) = \mathrm{MP}(\theta) \quad \ldots \ldots \quad (1)$$

$$\mathrm{MP}(-\theta) = -\mathrm{MP}(\theta) \quad \ldots \ldots \ldots \quad (2)$$

If

$$\tan x = \sin \theta,$$

then will

$$\mathrm{MP}(2x) = 2\mathrm{MP}(\theta) \quad \ldots \ldots \ldots \quad (3)$$

If

$$\tan a = \tan b \tan c \ldots$$

then will

$$\mathrm{MP}(2a - 90^{\circ}) = \mathrm{MP}(2b - 90^{\circ}) + \mathrm{MP}(2c - 90^{\circ}) \quad . \quad (4)$$

With regard to (1) it may be stated that from the form of the expression

$$\text{MP for lat } l^{\circ} = r \log e \tan \left(45^{\circ} + \frac{l^{\circ}}{2}\right),$$

the meridional parts in the first instance have reference to angles in the first quadrant only. The lemma enables us to pass to angles in the second quadrant.

Similarly by lemma (2) we can introduce negative angles also.

The result involved in (3) is exceedingly important, for it follows from this that if we have a logarithmic formula connecting the sines and cosines of parts of a spherical triangle, we may pass by means of auxiliary angles to other logarithmic formulæ, involving only the meridional parts of the angles employed, and that not only for right-angled and quadrantal triangles, as in the *Nautical Magazine*, but for any spherical triangle whatever.

As an example we may take one of the family of formulæ which express a function of an angle of a spherical triangle in terms of functions of the sides, supposed known. These expressions are perhaps, from a navigator's point of view, the most important which spherical trigonometry presents; for in the problem of finding the hour angle of a body, and thence the longitude of the place, such a formula may have to be brought into requisition on board a fast steam-ship as many as four or five times in the course of twenty-four hours. And while many of the problems of navigation may be, to some extent, "dodged" or evaded by the use of some of the many tables which ingenious persons have devised, there is no getting away from the hour-angle problem, because in that case the necessary degree of accuracy is more minute than any table of reasonable size could be expected to afford, unless we are content to spend more time and trouble in interpolating for variations in the values of the elements from the arguments given in the tables, than would suffice for the actual calculation by logarithms.

Let us assume that in the spherical triangle A B C we have to deal with the expression

$$\tan \frac{A}{2} = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}}.$$

Assume that

$$\sin (s-b) = \tan x \qquad \sin s = \tan w,$$
$$\sin (s-c) = \tan y \qquad \sin (s-a) = \tan z.$$

So that

$$\tan \frac{A}{2} = \sqrt{\frac{\tan x \tan y}{\tan w \tan z}}.$$

By lemma (3) we have

$$\mathrm{MP}(2x) = 2\mathrm{MP}(s\text{-}b),$$

and so on for y, w, z; a system of equations which will determine $2x$, $2y$, $2w$, $2z$.

Then by lemma (4)

$$\mathrm{MP}(A - 90^{\circ}) = \tfrac{1}{2}\{\mathrm{MP}(2x - 90^{\circ}) + \mathrm{MP}(2y - 90^{\circ}) - \mathrm{MP}(2w - 90^{\circ}) - \mathrm{MP}(2z - 90^{\circ})\},$$

whence A is readily determined.

The formula here established is only given as an illustration of the ease with which by the aid of lemma (3) we may pass from a sine or cosine formula to one involving meridional parts only by the simplest possible transformations.

The processes deduced by M. Guyou from the curves of altitude upon the Mercator's chart are probably somewhat shorter, and more likely, therefore, to be adopted for general use. His methods of procedure however, although, as has been well said of them by an Italian critic, "of high scientific interest for their originality and rigorous analysis," may be found somewhat subtle and difficult to follow by any but expert mathematicians. At all events, although, as has been said, the Guyou formulæ were received in Italy with much favour, mathematicians in that country lost no time in setting to work to establish them upon a basis purely trigonometrical.

An interesting article in the *Rivista Marittima* (Rome) for January 1897, by Signor P. L. Cattolica, "Capitano di corvetta," gives a summary of the work done in 1896 by Signor Molfino and other writers, whence it appears that the principal Guyou formulæ may be deduced with little difficulty from the well-known Napier's analogies as follows.

Let us suppose, as before, that in a spherical triangle the three sides a, b, c being given, it is required to determine the angles A, B.

We have

$$\tan\frac{a+b}{2} = \frac{\cos\frac{A-B}{2}\tan\frac{c}{2}}{\cos\frac{A+B}{2}}$$

$$= \frac{\cos\frac{A}{2}\cos\frac{B}{2} + \sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2} - \sin\frac{A}{2}\sin\frac{B}{2}}\tan\frac{c}{2}$$

$$= \frac{1 + \tan\frac{A}{2}\tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}}\tan\frac{c}{2}.$$

Let

$$\tan\frac{A}{2}\tan\frac{B}{2} = \tan\frac{x}{2} \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

Then

$$\tan\frac{a+b}{2} = \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\tan\frac{c}{2} = \tan\left(45^\circ + \frac{x}{2}\right)\tan\frac{c}{2}.$$

Whence

$$MP(x) = MP(90^\circ - c) - MP(90^\circ - \overline{a+b}) \quad . \; . \quad (2)$$

An equation which determines x.

While from equation (1) it may be deduced that

$$MP(90^\circ - A) + MP(90^\circ - B) = MP(90^\circ - x) \quad . \quad (3)$$

Proceeding in the same manner to expand

$$\sin\frac{A+B}{2}, \sin\frac{A-B}{2}$$

in the expression

$$\tan\frac{a-b}{2} = \frac{\sin\frac{A-B}{2}}{\sin\frac{A+B}{2}}\tan\frac{c}{2},$$

and assuming that

$$\tan\frac{B}{2}\cot\frac{A}{2} = \tan\frac{y}{2} \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

we arrive at the equations

$$MP(y) = MP(90^\circ - \overline{a-b}) - MP(90^\circ - c) \quad . \; . \quad (5)$$

$$MP(90^\circ - B) - MP(90^\circ - A) = MP(90^\circ - y) \quad . \quad (6)$$

By adding and subtracting each side of the two equations (3) and (6), we obtain equations which will enable us to determine the values of A and B respectively.

In place of the notation "MP," M. Guyou adopts the Greek letter λ (lambda). Thus, meridional parts for an angle $\theta = \lambda(\theta)$.

He also indicates the meridional parts of the complement of an angle by the symbol Co-λ, so that meridional parts for the angle $(90^\circ - \theta) = \text{Co-}\lambda\,(\theta)$.

And in his excellent collection of tables the values of λ and Co-λ are given for each angle side by side, an arrangement which much facilitates the work of computation.

The ordinary employment of Napier's analogies in practical work is limited to finding the remaining two sides when two angles and the included side are given, or to finding the remaining angles when two sides and the included angle are known. It is a somewhat remarkable extension of their functions to find that they suffice also to furnish satisfactory logarithmic formulæ for solving a triangle where the three sides are the given parts. In a similar manner formulæ may be found which will determine the sides when the three angles are given, so that formulæ of the type which gives $\tan\frac{A}{2}$ in terms of functions of the sides, or $\tan\frac{a}{2}$ in terms of functions of the angles may be dispensed with altogether.

It would be premature at present to hazard a conjecture as to whether the new processes will come into general use in England. In these matters we move slowly. The British mariner does not easily surrender the methods upon which he has been brought up, the practice of which becomes almost automatic with him, and he looks with feelings of doubt, tempered with suspicion, upon any novelties that may be brought to his notice. But some advantages, at least, of a system of rules involving the use of only one table of logarithms must be obvious to all. In the first place, as has been already mentioned, we have that of the greater simplicity in the statement of rules, and the diminished risk of error through the taking out of a logarithm from a wrong column. But even more important than these is the saving of time lost at present in turning over the leaves of tables in hunting for sines and cosines in different parts of a somewhat bulky book. In the table of meridional parts we have but 5400 logarithms, occupying some nine pages of Inman's collection, not more than might be printed on a sheet of cardboard of moderate size, so as to save the turning over of leaves altogether.

These logarithms furnish results correct to the nearest minute of arc, which is the usual limit of accuracy aimed at by the practical navigator.

As the case stands at present, the new system is well thought of in France; it has excited considerable attention in Italy, and has won the approbation of at least one distinguished authority in Spain; so that, perhaps, M. Guyou is not over-sanguine in his expectation that "the table of meridional parts is destined to become sooner or later the universal instrument of computation amongst mariners."

H. B. G.

THE NEW PHYSICAL RESEARCH LABORATORY AT THE SORBONNE.

AN interesting account of the new physical laboratory at the Sorbonne recently appeared in *La Nature*.

This laboratory, originally situated in the old Sorbonne, was founded in 1868 by M. Jamin, who was its director until his death in 1886. In 1894 it was transferred to the new Faculty of Sciences, and was reconstructed by the architect M. Nénot. At the present time M. Lippmann, member of the Institute, is the director. Although this change took place in 1894, the work has only recently been carried on in the usual manner.

The new buildings are surrounded by other buildings connected with the Sorbonne, and are therefore away from any disturbances caused by passing vehicles. On the ground floor, after passing an entrance hall with a cloak-room, there is a large room (Fig. 1) two stories high, and measuring 16 metres (about 52 feet) long by 12 metres broad (about 39 feet). Six physicists can work here, provided their work does not require any special conditions with regard to light and isolation. In the middle of the room, and at the corners there are solid stone pillars isolated from the floor; a "comparateur" is attached to the one in the middle. Each of the six places has four jets of gas, two incandescent lamps, one arc lamp, and a water-tap. About two yards above each table there is a joist, thus making it possible