LETTERS TO THE EDITOR.

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On the Meaning of Symbols in Applied Algebra.

ON reading the correspondence in NATURE (vol. lv.) on this subject, my sympathies were with the physicists as typified by the Professors Lodge ; but I think that the mathematicians as typified by Messrs. Jackson and Cumming have a legitimate grievance.

The following statement is "abhorrent" to the mathematicians. The horizontal intensity of the earth's magnetic field at a certain point is

$$\cdot 200 \frac{\sqrt{gm.}}{sec. \sqrt{cm.}}$$

$$= \cdot 200 \frac{\sqrt{\cdot 00220 \ lb.}}{(min. /60) \sqrt{\cdot 0328 \ ft.}}$$

$$= 3 \cdot 11 \frac{\sqrt{lb.}}{min. \sqrt{ft.}}$$

The physicist attaches a definite-enough meaning to this statement, and to the result of this little piece of generalised arithmetic. This is that if an observer go through the well-known process of finding H with two different sets of instruments: (1) a balance with gramme weights, a clock counting in seconds, and a scale divided to centimetres ; and (2) a balance with lb. weights, a clock counting in minutes, and a scale divided to feet ; then if his results on reduction give H $=$ ·200 in the first case, they will give H $=$ 3·11 in the second.

The mathematician will not for a moment dispute this result, and he will not deny that precisely similar processes will always give correct results. But he is, nevertheless, inclined to take up the position that no meaning can be assigned to the combination $(gm.)^{\frac{1}{2}} sec.^{-1} (cm.)^{-\frac{1}{2}}$. And his legitimate grievance is that nobody has placed these convenient processes on a general logical basis. (I believe this last is a fact.)

There is nothing illogical or mathematically immoral in the following simple assertions. In ordinary algebra there is no meaning attached to a length × another length, or to a length ÷ a time. We may, therefore, assert that a length × another length *shall mean* a certain area, viz. that of a rectangle, two of whose adjacent sides are the lengths ; and a length ÷ a time *shall mean* the velocity of a body which covers the length in the time. We are at perfect liberty to make these definitions, even if it should turn out that the ordinary laws of algebra will not hold for the new kind of multiplication and division. But if, as it turns out is the case, those laws should hold, we have extended the meaning of algebraic results, which is a great gain ; and we have provided ourselves with a new physical instrument of thought, which is a greater gain.

How to put all such mathematical processes, which the physicist is constantly employing, on one general logical basis? The following definitions hint a sketch of one way of proceeding.

In the definitions "number" will be taken to mean any real algebraic quantity—positive or negative, rational or irrational.

The algebraic definition of variation is applicable equally to numbers and to physical quantities. Let A and B be either two numbers or two physical quantities, possibly of different kinds. The ordinary algebraic definition of variation may be expressed thus :—A $\propto$ B if A depends on B in such a way that when B is multiplied by *any* number, A is multiplied by the same number. For instance, if the base of a triangle be given the area $\propto$ the altitude. The first of the following definitions includes the above as a particular case.

Definition 1. *A $\propto$ B^n if A depends on B in such a way that when B is multiplied by any number x, A is multiplied by x^n.* For instance,

$$\text{Edge of cube} \propto (\text{volume})^{\frac{1}{3}} ;$$

and in a race over a given course

$$\text{Runner's average velocity} \propto (\text{his time})^{-1}.$$

Definition 2. *If X is determined by, and depends in a specified manner on, the independent physical quantities (or numbers) A, B, C, ..., in such a way that X $\propto$ A^a when B, C ... are constant, and X $\propto$ B^b when A, C, ... are constant, &c., then*

$$X = A^a B^b C^c \ldots$$

The words "in a specified manner" are important. For instance, an area can be made to depend on two independent lengths in an infinite variety of ways. The specified manner *might* be as follows :—The area X is the area of a triangle whose base is A and altitude B. Then according to the definition X $=$ A B. But this is not the conventional specification. For that of course we must read "rectangle" for "triangle." Again an acceleration X may be made to depend in the way described in the definition on an independent length A and time B as follows :—X is the acceleration with which a body must move from rest to describe the length A in the time B. According to the definition we should then have X $=$ A B^{-2}. But this again is not the conventional specification. For the latter we must read "X is half the acceleration," for "X is the acceleration."

With these definitions it is not hard to show (1) that *all the laws of ordinary algebra* which have any meaning under the new circumstances *are true*, and (2) that *all such laws are true generalisations of the ordinary laws* in that the latter are particular cases.   ALEX. McAULAY.

University of Tasmania, Hobart, June 19.

Dog Running on Two Legs.

THE following instance shows how easily and well a four-legged animal can adapt itself to run on two legs only.

Last July a beautiful black and white shepherd's dog, on the Downs farm, near here, was caught amongst the knives of a reaping machine. Both the legs on the dog's right side were dreadfully mangled, and the animal almost bled to death. The right hind leg was so torn that one long piece and several small pieces of bone dropped from the wound. The dog lay for some time senseless and practically bloodless and lifeless. The kind-hearted shepherd, however, to whom the dog belonged, would not allow it to be at once destroyed ; he bound up its terrible wounds, put it carefully in a wheelbarrow, wheeled it home, and nursed it. After two or three weeks the animal had so far recovered as to be able to crawl and move about on its two left legs with a little assistance from its crushed right fore-leg.

This dog now lives with the shepherd at Dunstable, and runs backwards and forwards to Downs farm—a mile off—every day. The greater part of the journey is performed on the two legs of its left side, as the dog can do nothing whatever with its right hind-leg, and the right fore-leg is so damaged as to be only useful as a slight occasional prop. In starting to run, the dog quickly gets up, jerks his ruined right fore-leg over the left leg, balances itself on its two left legs only, and very rapidly hops off in the style of a large agile bird, both right legs hanging useless. With this strange mode of rapid progression it now attends to sheep exactly in the way of an ordinary uninjured dog. It is a most affectionate animal, and is now apparently full of life and health. When I went to see it this morning, it sprang up and happily bounded to me balanced on its two left legs.   WORTHINGTON G. SMITH.

Dunstable.

Foraminifera in the Upper Cambrian of the Malverns.

IN the early part of this year, whilst engaged in researches in the *Sphærophthalmus* zone of the Upper Lingula Flag Series, Prof. Theodore Groom, of Cirencester, found a shaley limestone which, when examined superficially under a fairly high power, showed indications of Foraminifera. Dr. Groom had a thin section prepared from this rock, and detected in it undoubted remains of Foraminifera. This preparation, together with specimens of the rock, he has courteously placed in my hands for further investigation, the results of which will be embodied in an appendix to Dr. Groom's paper on these beds.

The Foraminifera, for the most part, belong to the genus *Spirillina*, which has hitherto never been found below Jurassic strata, and these organisms make up at least 20 per cent. of the bulk of some specimens of the rock. The other genera present appear to be *Lagena, Nodosaria (Dentalina), Marginulina,* and *Cristellaria.*