B. Todd, Treasurer Roy. Soc., Mr. R. Trimen, Prof. Unwin, Prof. Van't Hoff, Gen. Walker, Prof. Weiss, Mr. C. Welch, Dr. Wynne.

Sir Joseph Lister, in giving the toast of "Science in all Lands," remarked that it would be impertinent in such company to dwell on the advantages which science conferred upon humanity or upon the pleasures which she gave to those who had the privilege of cultivating her various branches. They were agreed that if the mighty project upon which the conference had met was brought to a successful issue it would very greatly promote the advance of science.

The toast was responded to by the Italian Ambassador (General Ferrero), who said that England had always taken a leading, sometimes the first, place in science from the days of Newton to those of Lord Kelvin, and the Royal Society had worthily represented the nation in its work for the advancement of science.

Prof. Mach also responded, remarking that men of science recognised no distinction of race or nationality, and they were all glad to co-operate with Englishmen in a work in which all men of science were interested, especially as the work was done under the auspices of the Royal Society.

Dr. Billings proposed "Success to the Conference and the Catalogue" in a humorous speech. He suspected that classification began in the Ark. Science was now getting so large and various that the projected summary would be of extreme value; but he did not quite know to what it would lead. If their object in carrying out this catalogue were achieved, they might anticipate a time when men and things and thoughts also would be catalogued. They might look forward down the vista of years to the time when a stranger in Hyde Park would see a passer-by with such a number as 26·053, and would then at once appreciate his status in every respect, and when the novelist would proudly show that his heroine had twenty-six points in her character, while a rival writer had only achieved nineteen.

Prof. Darboux, Prof. Möbius, and Prof. Forel briefly acknowledged the toast.

The Treasurer of the Royal Society (Sir John Evans) proposed "The Guests," and expressed the hope that the deliberations of the conference would be ultimately successful.

Sir Donald Smith, High Commissioner for Canada, responded.

The Belgian Minister proposed "The Royal Society," which he said, was the mother and model of all similar societies in Europe, and was based on the principle that science knew nothing of nationality. The president was a great master of antiseptic surgery; if he could only introduce the principles by which he was so distinguished into the realm of politics and international relations he would be one of the greatest benefactors of the human race.

The President, in response, said the society was proud to take the lead in so important a work as that of the Conference. It had given him personally much satisfaction to learn that the Conference on the first day had been exceedingly successful, and there was no doubt that if this movement was carried out, as they hoped it would be, it would prove of great help to science in all its branches.

ON THE MOTION OF A HETEROGENEOUS LIQUID, COMMENCING FROM REST WITH A GIVEN MOTION OF ITS BOUNDARY.[1]

I USE the word "liquid" for brevity to denote an incompressible fluid, viscid or inviscid, but inviscid unless the contrary is expressly stated. A finite portion of liquid, viscid or inviscid, being given at rest, within a bounding vessel of any shape, whether simply or multiply continuous; let any motion be *suddenly* produced in some part of the boundary, or throughout the boundary, subject only to the enforced condition of unchanging volume. Every particle of the liquid will instantaneously commence moving with the determinate velocity and in the determinate direction, such that the kinetic energy of the whole is less than that of any other motion which the liquid could have with the given motion of its boundary.[1] This proposition is true also for an incompressible elastic solid, manifestly; (and for the ideal "ether" of *Proc. R.S.E.*, March 7, 1890; and Art. xcix. vol. iii. of my Collected Mathematical and Physical Papers). The truth of the proposition for the case of a viscous liquid is very important in practical hydraulics. As an example of its application to inviscid and viscous fluid and to elastic solid consider an elastic jelly standing in an open rigid mould, and equal bulks of water and of an inviscid liquid in two vessels equal and similar to it. Give equal sudden motions to the three containing vessels: the instantaneous motions of the three contained substances will be the same. Take, as a particular case, a figure of revolution with its axis vertical for the containing vessel and let the given motion be rotation round this axis suddenly commenced and afterwards maintained with uniform angular velocity. The initial kinetic energy will be zero for each of the three substances. The inviscid liquid will remain for ever at rest; the water will acquire motion according to the Fourier law of diffusion of which we know something for this case by observation of the result of giving an approximately uniform angular motion round the vertical axis to a cup of tea initially at rest. The jelly will acquire laminar wave motion proceeding inwards from the boundary. But in the present communication we confine our attention to the case of inviscid liquid.

The now well-known solution[2] of the minimum problem thus presented, when the bounding surface is simply continuous, is, simply: that the initial motion of the liquid is irrotational. That the *initial* motion *must be irrotational*[3] is indeed obvious, when we consider that the impulsive pressure by which any portion of the liquid is set in motion is everywhere perpendicular to the interface between it and the contiguous matter around it, and therefore the initial moment of momentum round any diameter of every spherical portion, large or small, is zero. But that irrotationality of the motion of every spherical portion of the liquid suffices to determine the motion within a simply continuous boundary having any stated motion, is not obvious without mathematical investigation.

Whether the boundary is simply continuous, or multiply continuous, irrotationality suffices to determine the motion produced, as we now suppose it to be produced, from rest by a given motion of the boundary.

Now in a homogeneous liquid acted on by no bodily force, or only by such force (gravity, for example) as could not move it when its boundary is fixed, the motion started from rest by any movement of the boundary remains always irrotational, as we know from elementary hydrokinetics. Hence, if at any time the boundary is suddenly or gradually brought to rest, the motion of every particle of the liquid is brought to rest at the same instant. But it is not so with a heterogeneous liquid. Of the following conclusions Nos. (1), (2), (3) need no proof. To prove

[1] Read at the Royal Society of Edinburgh, by Lord Kelvin, on April 6.

[1] *Cambridge and Dublin Mathematical Journal*, February 1849. This is only a particular case of a general kinetic theorem for any material system whatever, communicated to the Royal Society, Edinburgh, April 6, 1863, without proof (*Proceedings*, 1862-63, p. 114), and proved in Thomson and Tait's "Natural Philosophy," sec. 317, with several examples. Mutual forces between the containing vessel and the liquid or elastic solid, such as are called into play by viscosity, elasticity, hesivity (or resistance to sliding between solid and solid), cannot modify the conclusion, and do not enter into the equations used in the demonstration.

[2] Thomson and Tait's "Natural Philosophy," sec. 312.

[3] That is to say, motion such that the moment of momentum of every spherical portion, large or small, is zero round every diameter.

No. (4) remark that as long as there is any motion of the heterogeneous liquid within the imperfectly elastic vessel the liquid must be losing energy; and the energy cannot become infinitely small with any finite spherical portion of the liquid homogeneous.

(1) The initial motion of a heterogeneous liquid is irrotational only at the first instant after being *quite suddenly* started from rest by motion of its boundary. Whatever motion be subsequently given to the boundary the motion of the liquid is never again irrotational. Hence

(2) If the boundary be suddenly brought to rest at any time, the liquid, unless homogeneous throughout, is not thereby brought to rest; and it would go on for ever with undiminished energy if the liquid were perfectly inviscid and the boundary absolutely fixed. The ultimate condition of the liquid, if there is no *positive* surface tension in the interfaces between heterogeneous portions, is an infinitely fine mixture of the heterogeneous parts.[1] And, if there were no gravity or other bodily force acting on the liquid, the density would ultimately become uniform throughout. Take, for example, a corked bottle half full of water or other liquid with air above it given at rest. Move the bottle and bring it to rest again: the liquid will remain shaking for some time. An ordinary non-scientific person will scarcely thank us for this result of our mathematical theory. But, when we tell him that if air and the liquid were both perfectly fluid (that is to say perfectly free from viscosity), the well-known shaking of the liquid surface would, after a little time, give rise to spherules tossed up from the main body of the liquid; and that the shaking of the liquid, left to itself in the bottle supposed perfectly rigid, will end in spindrift of spherules which would be infinitely fine if the capillary tension of the interface between liquid and air were infinitely small, he may be incredulous unless he tends to have faith in all assertions made in the name of science.

(3) If the boundary is an enclosing vessel of any real material (and therefore neither perfectly rigid nor perfectly elastic), and if it is laid on a table and left to itself, under the influence of gravity, the liquid, supposed perfectly inviscid, will lose energy continually by generation of heat in the containing vessel, and will come asymptotically to rest in the configuration of stable equilibrium with surfaces of equal density horizontal and increasing density downwards.

(4) With other conditions as in (3), but no gravity, the ultimate configuration of rest will be infinitely fine mixture (probably, I think of equal density throughout). Consider, for example, two homogeneous liquids of different densities filling the closed vessel, or a single homogeneous liquid not filling it. As an illustration, take a bottle half full of water, and shake it violently. Observe how you get the whole bottle full of a mixture of fine bubbles of air, nearly homogeneous throughout. Think what the result would be if there were no gravity, and if the water and air were inviscid and the bottle shaken as gently as you please; and if there were perfect vacuum in place of the air; or, if for air were substituted any liquid of density different from that of water.

[1] "Popular Lectures and Addresses," by Lord Kelvin, vol. i. pp. 19, 20, and 53, 54. See also *Philosophical Magazine*, 1887, second half-year: "On the formation of coreless vortices by the motion of a solid through an inviscid incompressible fluid"; "On the stability of steady and of periodic fluid motion"; "On maximum and minimum energy in vortex motion."

THE RETURN OF BROOKS'S COMET.

ON July 6, 1889, Mr. W. R. Brooks, of Geneva, New York, U.S.A., discovered a somewhat faint, telescopic comet at R.A. 356°, Dec. 9° south, in the southern region of Pisces. It had a short spreading tail, and was moving slowly to the E.N.E.

Observations in a few days enabled the orbit to be computed, and the small inclination (6°) intimated that the comet was probably one of short period. This proved to be the case after further observation, and the time of revolution was determined as about seven years. Otto Knopf, from three positions obtained at Mount Hamilton on July 8, at Dresden July 30, and at Vienna on August 19, deduced the period as 7·286 years. The comet was followed until January 1890, and from the whole series of observations Prof. S. C. Chandler found a period of 7·073 years, and that the orbit at aphelion approaches very closely to the orbit of the planet Jupiter. From March to July 1886, the distance of the comet and planet appears to have been less than 10,000,000 miles. The theory was suggested by Prof. Chandler that the comet may be identical with Messier-Lexell's comet of 1770; but Dr. C. L. Poor, on reinvestigating the matter, found little evidence in support of the idea.

The possible connection of the comet with that of 1770 is by no means the only interesting feature of this object. On August 1, 1889, Prof. E. E. Barnard observed that the comet was broken up into several detached fragments. It had previously been seen single, and had been submitted to pretty general observation without anything remarkable having been detected; but on the night of August 1, it appeared to have been suddenly shattered by some extraordinary forces or vicissitudes of a very mysterious character. One of the smaller fragments, together with the largest mass, remained visible for several months, moving in concentric paths, and forming a very interesting and rare telescopic spectacle.

The comet was a fairly conspicuous object in telescopes, but it was not visible to the unaided eye. Its apparent motion was very slow, for early in November its position was only seven degrees north of the place it had occupied four months before.

Dr. Poor fixed the next perihelion passage for November 4, 1896, and an ephemeris was prepared by Bauschinger for the spring and summer of 1896, as it was expected the comet might be picked up some months before its arrival at perihelion. This expectation has been fully realised, for the comet was re-discovered on the night of June 20 by M. Javelle, using the 30-inch refractor of the observatory at Nice. Its place was almost identical with that given in the ephemeris, and the re-discovery of the comet may therefore be regarded as another triumph for mathematical astronomy.

This comet should prove an extremely interesting object in regard to its physical appearance and changes of aspect. At the present time it is in Aquarius a little west of *Delta* in that constellation, and its position during the next few weeks will be nearly stationary. The ephemeris by Bauschinger is as follows:—

1896.	R.A. h. m. s.	Decl. ° ′ ″	Log. Δ	Brightness.
July 15	22 39 1	−18 9 53	0·1124	1·14
19	39 58	12 28	0·0992	1·22
23	39 44	16 28	0·0866	1·31
27	39 24	21 49	0·0746	1·40
31	38 38	27 52	0·0633	1·50
Aug. 4	37 26	34 44	0·0529	1·59
8	35 50	41 51	0·0436	1·68
12	33 51	48 48	0·0353	1·76
16	22 31 34	−18 55 2	0·0284	1·84

Thus the comet is likely to be visible throughout the present summer and ensuing autumn, for its brightness is gradually increasing, and it will remain in a favourable position all the time. Its southern declination of more than 18° is, however, rather unfortunate, as its altitude is only about 20°, so that observers will require to watch it from a position commanding a good open view of the southern sky.

W. F. DENNING.