point out to the charitably disposed that there are a number of desiderata: there are, for example, no specimens of either the African or the American "Fin-foots."

LETTERS TO THE EDITOR.

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The Line Spectra of the Elements.

THE proper replies to Prof. Runge's letter in last week's issue of NATURE are three in number: viz. (1) that, as I pointed out in my former letter (NATURE of May 12, p. 29), the reasoning in my paper is valid if, as I there proved and as Prof. Runge now admits in the first sentence of his letter, Fourier's theorem can be applied to motions which approximate to non-periodic motions in any assigned degree and for any assigned time; (2) that I am not aware of anything I have written which countenances Prof. Runge's supposition that "Prof. Stoney has not noticed that a distinct property of the function is wanted in order to get a proper" [rather, a mathematically accurate] "resolution into a sum of circular functions"; and (3) that Prof. Runge is mistaken when he supposes that "the amplitudes and periods" [or frequencies] "of the single terms . . . do not approach definite values when the interval" [*i.e.* the periodic time of the recurrence required by Fourier's theorem] "increases indefinitely."

What the true state of the case is, is most easily shown as regards the frequencies of the lines; and as the proof is, I believe, new, and leads to a result of importance in the interpretation of spectra, I subjoin it.

Take a motion of the electron—

$$x = \text{The sum of partials such as} \left(a \sin \frac{2\pi kt}{j} + b \cos \frac{2\pi kt}{j}\right); \quad (1)$$

with similar expressions for the other two co-ordinates; in which the oscillation-frequencies, the k's, may be commensurable with one another, or incommensurable. If incommensurable, the motion is non-recurrent. Let this motion be arrested at intervals of T, and immediately started afresh as at the beginning. We thus obtain a recurrent motion consisting of a certain section of the motion (1) repeated over and over again. This new motion can be analyzed by Fourier's theorem, and we have to inquire what we thus obtain. Without losing anything in generality, we may confine our attention to the motion parallel to the axis of x, and to the single partial of that motion which is written out above, as all the partials lead to similar results.

Let us then examine by Fourier's method the motion which is represented by the equation—

$$x_k = a \sin \frac{2\pi kt}{j} + b \cos \frac{2\pi kt}{j} \quad . \; . \; . \; . \; . \; (2, a)$$

from $t = 0$ till $t = \text{T}$, and which is repeated from that on at intervals of T. If T is a multiple of j/k, Fourier's theorem simply furnishes equation $(2, a)$ as the complete expression for all time of the motion; so that in this case it indicates the same definite line in the spectrum as is furnished by the original partial of equation (1).

If T is not a multiple of j/k,

$$\text{T will} = (m + a)\frac{j}{k},$$

where m is a whole number and a a proper fraction. Equation $(2, a)$ then becomes

$$x_k = a \sin \frac{2\pi(m+a)t}{\text{T}} + b \cos \frac{2\pi(m+a)t}{\text{T}} \quad . \; . \; (2, b)$$

which is true from $t = 0$ till $t = \text{T}$, after which the motion is to be repeated. Then, by Fourier's theorem—

$$\left.\begin{aligned} x_k = \text{A}_0 + \text{A}_1 \sin \frac{2\pi t}{\text{T}} + \text{A}_2 \sin \frac{4\pi t}{\text{T}} + \; . \; . \; . \\ + \text{B}_1 \sin \frac{2\pi t}{\text{T}} + \text{B}_2 \sin \frac{4\pi t}{\text{T}} + \; . \; . \; . \end{aligned}\right\} (3)$$

is true of this motion for all time, in which

$$\text{A}_n . \int_0^{\text{T}} \sin^2 \frac{2\pi nt}{\text{T}} . \, dt = a \int_0^{\text{T}} \sin \frac{2\pi(m+a)t}{\text{T}} . \sin \frac{2\pi nt}{\text{T}} . \, dt + b \int_0^{\text{T}} \cos \frac{2\pi(m+a)t}{\text{T}} . \sin \frac{2\pi nt}{\text{T}} . \, dt;$$

$$\text{B}_n . \int_0^{\text{T}} \cos^2 \frac{2\pi nt}{\text{T}} . \, dt = a \int_0^{\text{T}} \sin \frac{2\pi(m+a)t}{\text{T}} . \cos \frac{2\pi nt}{\text{T}} . \, dt + b \int_0^{\text{T}} \cos \frac{2\pi(m+a)t}{\text{T}} . \cos \frac{2\pi nt}{\text{T}} . \, dt;$$

which, when integrated, give the following values for A_n and B_n—

$$\left.\begin{aligned} \text{A}_n &= \frac{a \sin 2\pi a - b(1 - \cos 2\pi a)}{2\pi}\left(\frac{1}{d} - \frac{1}{s}\right) \\ \text{B}_n &= \frac{a(1 - \cos 2\pi a) + b \sin 2\pi a}{2\pi}\left(\frac{1}{d} + \frac{1}{s}\right) \end{aligned}\right\} \; . \; . \; . \; (4)$$

where d stands for $(m - n + a)$, and s for $(m + n + a)$.

This furnishes a very remarkable spectrum, a spectrum of lines that are equidistant on a map of oscillation-frequencies, and that extend over the whole spectrum. But they are of very unequal intensities. If T is a long period, m is a high number. The lines are then ruled close to one another, and their intensities are insensible except when n is nearly equal to m, the two brightest lines being the next to the position of the original line of equation (1), one on either side of it, and the others falling off rapidly in brightness in both directions.

If we take a longer period for T, m becomes a still higher number; the lines are more closely ruled and are more suddenly bright up to those on either side of the position of the original line of equation (1), to which also they are now closer; so that, at the limit, when T increases indefinitely, equation (3) becomes a mathematical representation of the original line of equation (1).

This interesting investigation is all the more important as it gives a clue to how rulings of lines which are equidistant and brighter up to the middle may arise; and I feel sure that Prof. Runge will join me in not regretting that he expressed the doubts which led to its solution.

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9 Palmerston Park, Dublin, June 3.

Stone Circles, the Sun, and the Stars.

ARTICLES by Mr. Norman Lockyer and Mr. Penrose, recently published in NATURE, have dealt with the positions of ancient Egyptian and Greek temples with relation to the rising sun, and to the pole star, or some star or stars in its vicinity. For some years past I have endeavoured to show, in papers read before the British Association and other Societies, that our stone circles had a relation to the rising sun, indicated usually by an outlying stone or by a notable hill-top in the direction in which the sunrise would be seen from the circle, and I have in some cases found similar indications towards the north, which may have referred to the pole or other northern star or stars. A paper containing many details as to these cases will shortly appear in the Journal of the Royal Archæological Institute.

There are six circles on Bodmin Moors, which at first sight appear to have no relation to each other, but which, if the 6-inch Ordnance map is to be relied upon, would seem to have been arranged on a definite plan (see accompanying plan).

The Stannon and Fernacre Circles are in line 1° (true) north of east with the highest point of Brown Willy, the highest hill in Cornwall; and the Stripple Stones and Fernacre Circles are in line with the summits of Garrow and Rough Tor, at right angles with the other line—namely, 1° west of (true) north. A line from the Trippet Stones Circle to the summit of Rough Tor would also pass through the centre of one of the Leaze Circles (about 12° east from true north). Other hills are in the direction of the rising sun. The Trippet Stones are 11½° south of west from the Stripple Stones, 10° east of south from the Stannon Circle, and about 13° west of south from the Fernacre Circle. The respective bearings of the other circles have already been given, and all are true (not magnetic) according to the 6-inch Ordnance map.