Such a comparison I have endeavoured to make, or rather to indicate the basis on which it may be made, so far as systems of geometrical algebra are concerned. As a contribution to analysis in general, I suppose that there is no question that Grassmann's system is of indefinitely greater extension, having no limitation to any particular number of dimensions.

J. WILLARD GIBBS.

The Flying to Pieces of a Whirling Ring.

IN NATURE of May 14 (p. 31) I notice a letter by Mr. C. A. Carus-Wilson on the rotation of a hollow steel flask, composed apparently of a spherical shell mounted on an axis constituting a diameter. Mr. Carus-Wilson speaks of this body as being under a "tension" of "31·5 tons per square inch" at a certain speed of rotation. He does not, however, specify what is the tension to which he refers, nor where it is found, neither does he give the density and elastic constants of the material nor indicate the method by which he arrived at his result.

So far as I know, the only problem of the kind which has yet been solved is that of an isotropic spherical shell [1] rotating about an imaginary axis through its centre at speeds at which the strains follow Hooke's law. This differs from the case Mr. Carus-Wilson speaks of, inasmuch as the existence of a real material axis must introduce conditions somewhat different from those assumed by the mathematical theory, and further the results obtained by this theory cannot legitimately be applied to speeds exceeding that where bulging becomes sensible, if indeed so far.

This solution is probably, however, the nearest to the practical problem at present attainable.

According to it the strains and stresses vary throughout the shell with the distance from the centre, and the angular distance from the axis of rotation. They also depend on the density and on the elastic properties of the material. There are also at every point three principal stresses, whereof one it is true vanishes over the surfaces. Thus such a statement as Mr. Carus-Wilson's requires further explanation.

According to the two theories most commonly entertained, the quantity which determines the limiting safe speed is the maximum value of either the *greatest strain* or the *maximum stress-difference*,—*i.e.* the algebraical difference between the greatest and least principal stresses at a point. Over the surfaces of the shell the absolutely greatest values of both these quantities are found, for shells of all degrees of thickness, in the equatorial plane—or plane through the centre perpendicular to the axis of rotation.

Denoting the angular velocity by ω, the radii of the outer and inner surfaces respectively by a and a', the density by ρ, Young's modulus by E, the greatest strain by s, the maximum stress-difference by S, and the stress at right angles to the meridian plane by Φ, the three last quantities being measured in the equator, the following are some of the results I found for materials in which Poisson's ratio is 1/4:—

	$Es/\omega^2\rho a^2$		$S/\omega^2\rho a^2$		$\Phi/\omega^2\rho a^2$	
	Inner surface.	Outer surface.	Inner surface.	Outer surface.	Inner surface.	Outer surface.
$a'/a = 0{\cdot}9$	0·950	0·833	1·064	0·866	0·912	0·866
$\frac{a-a'}{a}$ negligible	1·0	1·0	1·0	1·0	1·0	1·0

Apparently in the case mentioned by Mr. Carus-Wilson, $a'/a = 15/16 = 0{\cdot}9375$. Supposing the material to have Poisson's ratio = 1/4, which seems to accord fairly with experiments on steel, the approximate values of s, S, and Φ, for this case could be obtained by interpolation from those I give above. The differences between the values of corresponding strains and stresses at the two surfaces are less, of course, for $a'/a = 15/16$ than for $a'/a = 0{\cdot}9$, but still are far from negligible. Mr. Carus-Wilson's numerical result rather suggests that the tension he refers to is the stress Φ, measured as above in the equator, and that he employed the formula $\Phi = \omega^2\rho a^2$. This formula (see Cambridge Philosophical Transactions, vol. xiv. p. 300), is correct for the value of Φ in the *equator* in an *infinitely thin* shell, but it does not strictly apply to any shell whose thickness is comparable with its radius. In the paper in the Cambridge Transactions first referred to, there are given tables of the numerical measures of the strains and stresses over the surfaces for a series of values of a'/a for materials in which Poisson's ratio is 1/4. These give by interpolation fairly accurate values for all values of a'/a. For other values of Poisson's ratio, recourse must be had to the general formulæ given in the paper, unless $\epsilon, \equiv 1 - a'/a$, is very small, when the greatest values of s and S are given approximately by $Es/\omega^2\rho a^2 = 1 - \frac{1}{2}\epsilon(1 - \eta)$, $S/\omega^2\rho a^2 = 1 + \epsilon/(1 + \eta)$, where η is Poisson's ratio (see Camb. Trans., vol. xiv. p. 304).

May 16. C. CHREE.

[1] Cambridge Philosophical Society's Transactions, vol. xiv. pp. 467–483.

A Comet observed from Sunrise to Noon.

A SHORT time ago I got the loan of an old number of *Harper's Monthly* (March 1889), good reading matter being very acceptable, however old, in this outlandish place, in which I read an article, on the origin of celestial species, by J. Norman Lockyer, F.R.S., Cor. Inst. France, that set me thinking of what I observed of the great comet of 1882, when it made its tremendous plunge round the sun, on September 18. At that time I was master of a small vessel, trading in the Society Islands; and on the day mentioned—in latitude 16° 25′ S., longitude 151° 57′ W. of Greenwich, a position about midway between the two islands Bolabola and Maupiti (the Maurua of Cook)—I saw, with the naked eye, the comet travel about 90° of the circle of the sun's disk, between sunrise and noon; but what made it most remarkable to us was that it should be possible for us, in a perfectly clear sky, to be able to watch it all, from sunrise to noon, with very little more distress to the eye than if in a clear night looking at a full moon.

Now, Sir, may it not be that this is partly a *proof* of the theory set forth by Norman Lockyer in the article above mentioned, viz. that comets are swarms of meteorites in collision, travelling through space, and that the outer invisible part of the swarm that formed this comet's nucleus had partially eclipsed the sun, like a veil over it? I am not aware if it was noticed by any competent astronomer or not, but the chances are that none had the splendid opportunity that we had to see the phenomena; so, Sir, knowing that men of science are always glad to get facts from observers in all parts of the world is my excuse for writing this to you, not knowing Mr. Lockyer's address. Thinking this, although late, may probably be of some interest to the scientific world, I leave you to do what you may think proper with it. WM. ELLACOTT.

Raiatea, January 30.

Graphic Daily Record of the Magnetic Declination or Variation of the Compass at Washington.

I BEG to call your attention to the enclosed reprint from the May Pilot Chart of curves of magnetic declination as recorded at the United States Naval Observatory at Washington. This reprint admits of reproduction more readily than the curves as shown on the Pilot Chart, being in black and white, and only reduced to two-fifths of true size (the reduction on the Pilot Chart itself being one-quarter). It will be interesting to this Office to elicit expressions of opinion relative to the advantages of the prompt publication of these curves. The experiment is to be tried for three months, but it is not likely to be continued longer unless certain decided advantages develop. It may be of sufficient interest to NATURE to republish these curves, and thus assist us in giving them wide publicity.

RICHARDSON CLOOER, Hydrographer.

Washington, D.C., May 6.

[We are unable to print the curves, but we may note that they are issued with the following explanation:—"These curves indicate graphically the true direction in which the magnetic needle at the Naval Observatory pointed during each instant from noon, March 29, to noon, April 30. The base-line shows a slight break at the end of each two hours, 75th meridian time, and the amount of westerly variation at any time is 4° *plus* the number of minutes represented by the height of the curve above the base line at that time, measured by the scale at the right or left margin of the diagram. The slight breaks in the curve itself occur when the chronograph sheets are changed. Although the daily change of variation at any one place, even in magnetic storms such as those that have occurred during the past month, is too small to be of any importance in practical navigation, yet it is thought that the prompt publication of these curves cannot fail to interest masters of vessels, as well as scientific men. The mean daily curve, which can be drawn by taking the average of many such curves, shows that there is a regular, though slight,