

Wolstenholme's and others are here obtained by the use of circular co-ordinates.—The reduction of

$$dx/\sqrt{A(1+mx^2)(1+nx^2)} \text{ to } Mdy/\sqrt{(1-y^2)(1-b^2y^2)}$$

by the substitution  $x^2 = a + by^2/a' + b'y^2$ , by H. P. Manning. A table of available forms is added, and attention drawn to those forms in it given by Cayley ("Elliptic Functions," p. 316).—A simple statement of proof of reciprocal theorem, by J. C. Field.—Related expressions for Bernouilli's and Euler's numbers, by J. C. Field.—In No. 2 appears a third memoir, on a new theory of symmetric functions, by Major P. A. MacMahon, R.A. Attention is drawn to a fundamental theorem in operations, given without proof. It is a generalization of a theorem by Sylvester which is itself a generalization of Taylor's theorem; "it enables us from any linear function P of the operators to determine another linear function Q, such that  $\exp. P = \exp. Q$ ," the bar in  $\exp. u$  being used by the author to indicate that the multiplication of operators that occur in  $u$  is symbolic.—M. Joseph Perrott also contributes a paper entitled "Remarque au sujet du théorème d'Euclide sur l'infinité du nombre des nombres premiers."

SOCIETIES AND ACADEMIES.  
LONDON.

Royal Society, April 9.—"The Measurement of the Power supplied by any Electric Current to any Circuit." By Prof. W. E. Ayrton, F.R.S., and W. E. Sumpner, D.Sc.

I.—During the meeting of the Electrical Congress in Paris in 1881, one of us<sup>1</sup> devised a method of using an electrometer for measuring the power given to any circuit by any current. This method is the only electrical one hitherto published, the accuracy of which does not depend on assumptions either as regards the character of the current variations or as regards the nature of the circuit the power given to some part of which we desire to measure.

In view then of the present wide use of alternating currents for industrial purposes, it might have been expected that this electrometer method of measuring the power given by any intermittent or alternating current to an inductive circuit would have been extensively employed. Unfortunately, however, as pointed out by one of us in conjunction with Prof. Perry (Journal of Soc. of Tel. Eng. and Elects., vol. xvii, 1888),

the use of this method is restricted by the fact that Sir W. Thomson's quadrant electrometers do not generally obey the mathematical law given for these instruments in text-books,<sup>1</sup> as it was supposed they did when this electrometer method of measuring power was first suggested. And hence the main result that has, up to the present time, followed from the publication of this method has been the stimulation of inventive minds to devise forms of electrometers in which the text-book law is strictly fulfilled.

In 1888, Mr. Blakesley published a very ingenious method for measuring the power supplied by alternating currents to the primary coil of a transformer, by the use of three dynamometers. The proof originally given was geometrical, and was based on several assumptions, amongst others that the currents and magnetic fluxes varied with the time according to a simple sine law. An analytical proof has recently been given (meeting of Physical Society, February 27, 1891) by one of us, in conjunction with Mr. Taylor, showing that the method gives equally good results however the currents and magnetic fluxes vary, but there still remains a serious objection to the method, as it is assumed that there is no magnetic leakage in the transformer, or, in other words, every line of force is supposed to thread each convolution of both primary and secondary coils. Further, the method cannot be used with a single circuit as the coils of one of the dynamometers must be placed in different circuits.

The employment of an electro-magnetic wattmeter for the measurement of power is well known, but errors are introduced when alternating currents are used, owing to the self-induction of the fine wire coil. Several investigators have considered the magnitude of this error, and have suggested various devices for reducing it to a minimum.

II.—Several months ago, however, while working at alternate-current interference, we noticed that it was possible to employ an extremely simple method for measuring the power supplied by any current to any circuit. This method, which has since been in regular use in the laboratories of the Central Institution, is quite independent of any assumptions as to the nature of the current, or of the circuit, the power given to which it is desired to measure, and it has the further great advantage that the only measuring instrument required is the ordinary alternate-current voltmeter of commerce.

In series with the circuit *ab* (Fig. 1), the power given to which we desire to measure, connect a non-inductive resistance *bc* of *r* ohms,

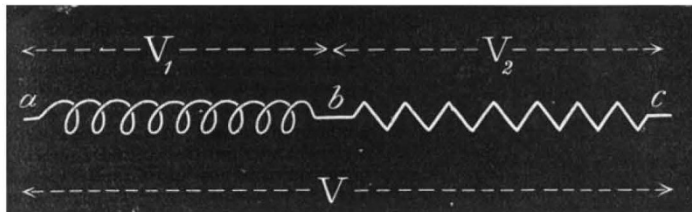


FIG. 1.

Let  $V_1$ ,  $V_2$ , and  $V$  be the readings of the voltmeter when applied between *a* and *b*, *b* and *c*, and *a* and *c* respectively; then, if  $W$  be the mean watts supplied to the circuit *ab*, we have in all cases, whatever the nature of the current, or of the circuit *ab*—

$$W = \frac{1}{2r}(V^2 - V_1^2 - V_2^2) \dots \dots \dots (1)$$

For, let  $v_1$ ,  $v_2$ , and  $v$  be the instantaneous values of the P.D. between *a* and *b*, *b* and *c*, and *a* and *c* at some moment *t*, then  $v = v_1 + v_2 \dots \dots \dots (2)$

If *a* be the current in amperes flowing through the circuit at time *t*, then  $av_1$  equals the watts *w* given to *ab* at that time. But

$$a = \frac{v_2}{r},$$

since the resistance *bc* is non-inductive;

$$\therefore w = \frac{v_1 v_2}{r}$$

Then, squaring (2), we have—

$$v^2 = v_1^2 + 2v_1 v_2 + v_2^2;$$

$$\therefore w = \frac{1}{2r}(v^2 - v_1^2 - v_2^2).$$

As this equation is true at every moment, it must also be true for the mean values of *w*,  $v^2$ ,  $v_1^2$ , and  $v_2^2$ .

So that

$$\int_0^T w dt = \frac{1}{2r} \left( \int_0^T v^2 dt - \int_0^T v_1^2 dt - \int_0^T v_2^2 dt \right),$$

and

$$W = \frac{1}{2r}(V^2 - V_1^2 - V_2^2),$$

which is the equation given above.

If the resistance of *bc* be not known, or if there be any fear that it may be changed by the passage of the current, then an

<sup>1</sup> We may mention that an investigation on quadrant electrometers has been going on from time to time at the Central Institution for the last five years, and we had hoped to have communicated the complete report long before this to the Royal Society.

ammeter (an alternate-current ammeter, of course, if alternate currents be employed) can be inserted in the circuit. Let the reading of this ammeter, which represents the square root of the mean square of the current be  $A$ , then, for  $r$  in (1) we may substitute  $V_2/A$ , or

$$W = \frac{A}{2V_2} (V^2 - V_1^2 - V_2^2) \dots \dots (3)$$

When employing this last formula, the non-inductive resistance  $bc$  may be that offered by incandescent lamps, since there is no objection to the resistance varying with different mean strengths of the current employed.

This voltmeter method of measuring power was arrived at quite independently of the electrometer method referred to above, but an examination of the electrometer method shows that it is practically equivalent to simultaneous measurements of three P.D.s.

An analysis of the equation (1) shows that the value of the non-inductive resistance  $r$ , which it is best to adopt in order to reduce to a minimum the error in  $W$  arising from errors in the voltmeter readings, is such that the potentials  $V_1$  and  $V_2$  are equal to each other. It can also be shown that the percentage error in estimating the power  $W$  due to errors in the voltmeter measurements arising either from faulty graduation of the scale, or from inaccurate readings, is from four to five times the percentage error of a single reading of the voltmeter.

As all instruments that are graduated for measuring the square root of the mean square of an alternating P.D., such as a hot-wire voltmeter, an electrostatic voltmeter, &c., really measure the mean square and not the square root of the mean square directly, it would be better, if such an instrument were to be employed for the method of measuring power described in this paper, that it should be graduated in mean squares of P.D.s. and not in the square roots of the mean squares. In that case the probable percentage error in the measurement of power by the method would be from 2 to 2.5 times the error in the measurement of each of the P.D.s.

It is, of course, clear that these errors to which we have been referring are not errors in any way essential to the method proposed for measuring power, since by the employment of an accurately graduated voltmeter, by exercising care in taking the readings, and, if necessary, by repeating the measurements two or three times and taking the means of the observations, the power can be measured to any degree of accuracy desired.

As in practical cases the sum of the two potentials  $V_1$  and  $V_2$ , will not often be much in excess of  $V$ , we may conveniently express the true power  $W$  in the following way.

If  $A$  is the current in  $ab$  (Fig. 1) as read by an alternating current ammeter, the apparent power absorbed by  $ab$  is

$$V_1 A.$$

The true power  $W$ , when  $V_2$  is made equal to  $V_1$ , can be shown to be

$$W = \left(1 - 2y + \frac{y^2}{2}\right) V_1 A,$$

where

$$y = \frac{V_1 + V_2 - V}{V_1},$$

or as  $y$  is usually a small number,

$$W = (1 - 2y) V_1 A.$$

Thus the error made in assuming that  $V_1 A$  represents the true power is 8 per cent. if  $(V_1 + V_2 - V)$  is 4 per cent. of  $V_1$ , also if, due to unsteadiness of the currents, or to error in the voltmeter readings, the value of  $(V_1 + V_2 - V)$  is uncertain to the extent of 1 per cent. of  $V_1$ , the uncertainty in estimating  $W$  is twice this, or 2 per cent.

III.—The method we have just described is well adapted for measuring the power supplied to an alternating current arc lamp, and is, moreover, likely to be of service in investigating the phenomena of the alternating current arc. Three of our senior students, Messrs. Kolkhorst, Thornton, and Weekes, have made a number of experiments on arc lamps by the use of this method, and from their experiments it would appear that the quality of the carbon employed affects materially the difference in phase between the currents passing through the arc and the P.D. between the carbons. If the arc be quite steady and only give out the rhythmic hum that accompanies a well-formed arc, such as can be obtained with cored carbons of good quality, the arc appears to act practically as a simple resistance, but if the arc

be maintained between uncored carbons of poor quality, and be hissing, there is considerable difference in phase between the current and the P.D. between the terminals; further, the experiments show that the current is very far from being a sine function of the time, although produced by a dynamo whose E.M.F. normally follows a harmonic law.

In addition to the difference of phase of P.D. and current that may be produced in the arc itself, there is the electro-magnet to be considered, by which the distance between the carbons is usually regulated in arc lamps. This electro-magnet will introduce lag between the P.D. at the terminals of the lamp and the current passing through the electro-magnet and the arc in series; and hence, even although the arc be perfectly steady, we find, even in the case of a Brush lamp especially intended for alternate currents, that the true power supplied to the electro-magnet and arc is 20 per cent. less than the product of the readings of the ammeter and the voltmeter attached to the lamp terminals, and which gives the square root of the mean product of the squares of the current and P.D.

If, however, the arc be between common carbons and be hissing, the difference, we find, is much greater. With cored carbons this Brush lamp requires a P.D. of about 35 volts to be maintained between its terminals, but if these cored carbons be replaced by common carbons and the arc be hissing, the P.D. between the terminals of the lamp at once rises to 45 or even 50 volts, although the current passing through the lamp and the amount of light given out remain practically as before. And then we find that the true power supplied to the lamp may be only one-half of the square root of the mean product of the squares of the current and P.D., so that the readings of the ammeter and voltmeter alone make the apparent power twice as great as the true power.

For the purpose of easily estimating the ratio of the true to the apparent power supplied, formula (3) may be thus written—

$$W = AV_1 \left\{ 1 - \frac{(V_1 + V_2 - V)(V_1 + V_2 + V)}{2V_1V_2} \right\} \dots (4)$$

from which we see that the expression in the brackets represents the ratio of the true to the apparent power supplied to the lamp or other circuit  $ab$  (Fig. 1). Hence the percentage error made in assuming that the power supplied to any circuit was the product of the ammeter and voltmeter readings would be in all cases, whatever the nature of the current or of the circuit,

$$100 \frac{(V_1 + V_2 - V)(V_1 + V_2 + V)}{2V_1V_2} \dots (5)$$

The following are samples of the results obtained with a hand-regulated lamp, there being no electro-magnet at all in series with

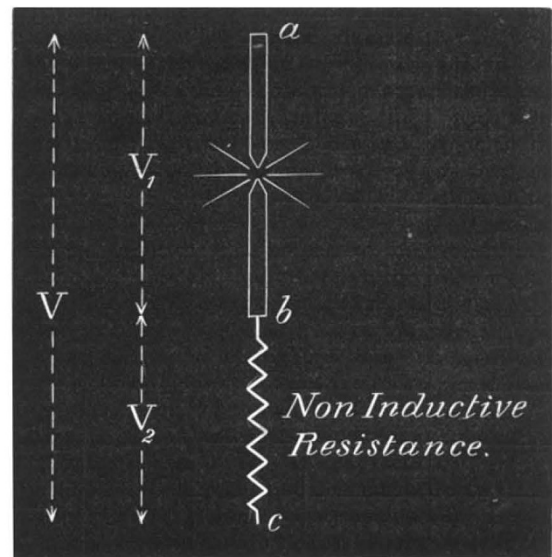


FIG. 2.

the arc (Fig. 2). The carbons were not cored, and the arc was hissing. The frequency was maintained at 200 periods per second.

TABLE I.

Square root of mean square			Of current in amperes.	Percentage error in estimating power formula (5).
Of P.D. in volts between				
$\alpha$ and $\delta$ . $V_1$ .	$\beta$ and $c$ . $V_2$ .	$\alpha$ and $c$ . $V$ .	A.	
55.0 45.4	66.0 75.4	108.0 107.3	12.3 11.8	24.0 45.8

For the purpose of obtaining an idea of  $\phi$ , the angle of phase difference produced by the hissing arc, between the current and the P.D., we may assume that the P.D. and current are sine functions of the time; then, as may be easily proved,

$$\cos \phi = \frac{V^2 - V_1^2 - V_2^2}{2V_1V_2} \dots \dots (6)$$

and the values of  $\phi$  for the two tests given above come out as  $40^\circ 20'$  and  $57^\circ 10'$ . It will, of course, be observed that this assumption of a harmonic law for the P.D. and current for the purpose of obtaining some idea of the value of  $\phi$ , in no way affects the generality of the method for the measurement of the power, since this is based on no such assumption.

The following are samples of the results obtained with a Brush alternate-current lamp regulated by an electro-magnet (Fig. 3),

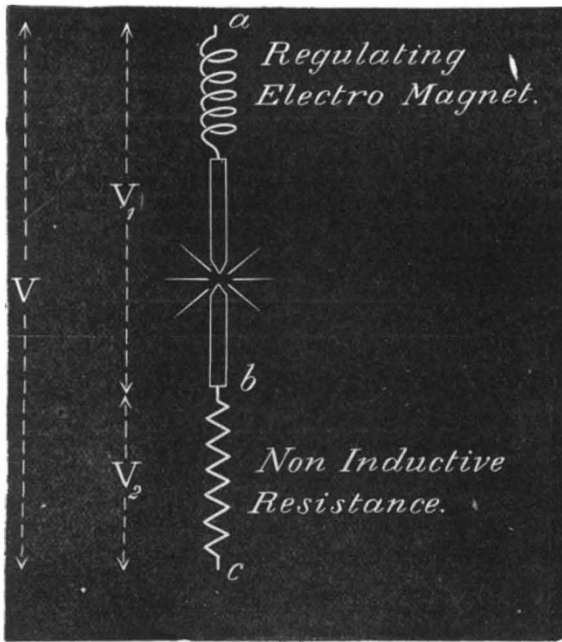


FIG. 3.

the carbons not being cored, and the arc hissing. The frequency was maintained at 200 periods per second.

TABLE II.

Square root of the mean square			Of current in amperes.	Percentage error in estimating power formula (5).	Lag between current and P.D. $\phi$ .
Of P.D. in volts between					
$\alpha$ and $\beta$ . $V_1$ .	$\beta$ and $c$ . $V_2$ .	$\alpha$ and $c$ . $V$ .			
64.8	58.0	108.4	13.0	44.0	$56 \frac{1}{2}$
59.8	64.2	107.4	12.0	50.5	$60 \frac{20}{100}$
55.0	67.3	107.4	10.6	47.0	$58 \frac{30}{100}$

The experiments already described tell us that a hissing arc may cause a considerable phase difference between the P.D. and the current, but they do not enable us to decide whether such an arc causes the current to lag behind the P.D., or to lead in front of it. To decide this point—that is, to decide whether a hissing arc acts like an inductive coil, or a condenser—a variety of experiments were made by putting induction or capacity in series with the arc. The following gives the result of one such experiment:—In series with a hand-regulated lamp (and, therefore, containing no electro-magnet), was placed a condenser of 89 microfarads (Fig. 4). Uncored carbons were used, and they were adjusted so that the arc was very short at first; the carbons were then not touched, and, as they burnt away, the arc grew longer and longer until it finally went out. The frequency was maintained at 200 periods per second.

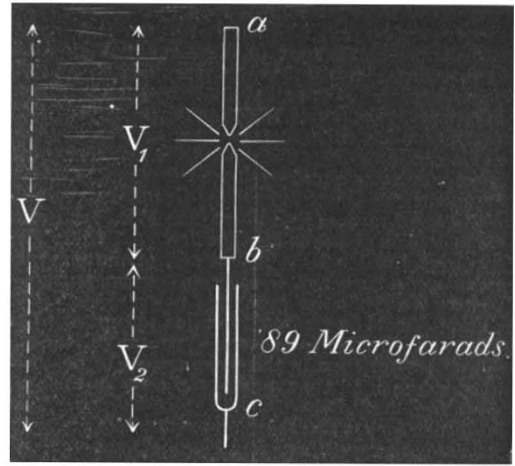


FIG. 4.

TABLE III.

E. M. F. of dynamo in volts.	Square root of mean square			Of current in amperes.	Sum of $V_1 + V_2$ .	Lag between current and P.D. $\phi$ .
	Of P.D. in volts between					
	$\alpha$ and $\beta$ . $V_1$ .	$\beta$ and $c$ . $V_2$ .	$\alpha$ and $c$ . $V$ .	A.		
59	35.4	89.0	72.3	12.0	124.4	0
	38.0	92.0	73.3	12.5	130.0	129
	51.2	104.5	74.3	14.0	155.7	133
	69.2	86.5	67.5	13.4	155.7	135

Comparing  $V$  with the E.M.F. of the dynamo, we see that the arc and the condenser together acted as a condenser on the whole; but, comparing  $V$  with  $V_1 + V_2$ , we see that the arc acted as an induction and not as a capacity.

Calculations from the measurements made on the assumptions that the arc acts like an induction coil, and that the current follows a simple sine law, show that the inductance of the arc itself is as great as that of the regulating electro-magnet used in the lamp. When, however, the inductances of the arc and electro-magnet in series is observed, it is found to be less than the sum of the two inductances. This shows conclusively that the current does not vary according to a simple sine law.]

Linnean Society, April 2.—Prof. Stewart, President, in the chair.—The Rev. Prof. Henslow exhibited specimens of *Oxalis cornuta*, Thunberg, a native of the Cape of Good Hope, and gave an interesting account of its introduction into the countries bordering the Mediterranean and the Canaries and Madeira, tracing its present northern distribution, so far as he had been able to ascertain it. A discussion followed, in which Messrs. A. W. Bennett C. B. Clarke, W. Bateson, and B. D.

Jackson took part.—Mr. A. B. Rendle, having examined the specimens of "Monchona" exhibited by Mr. Christy at a previous meeting, expressed the opinion that this trade product was the preserved fruit of a palm, belonging to a species apparently undescribed. It was stated somewhat vaguely by the importer to have come from the South Pacific.—Mr. Rendle also exhibited another specimen of an orange within an orange, which differed from that shown at a former meeting in that the inner orange possessed a rind, and was not entirely enveloped by the outer one.—The President exhibited an abnormal specimen of a butterfly (*Gonepteryx rhamni*) possessing five wings, or two hinder wings on one side.—Mr. W. Bateson then gave the substance of a paper by himself and Miss A. Bateson on variations in floral symmetry of certain plants with irregular corollas. He described the variations in number of parts and of symmetry occurring in the flowers of *Gladiolus*, *Veronica*, *Linaria*, and *Streptocarpus*, and showed that, although in these varieties there is considerable departure from the normal form, yet the resulting variety is often as definite as the normal form, and not less perfect in symmetry. It was suggested that the variations by which specific forms of symmetry are produced may also be thus distinct, and not of necessity involving transitional forms; and, for example, that the process by which the 4-petalled symmetry of *Veronica* arose from that of a 5-petalled ancestor was perhaps similar in kind to that by which the 3-petalled variety of *Veronica* is formed from the type, transitional forms being in such cases rare, or even absent. An interesting discussion followed, in which the President, Prof. Henslow, Messrs. C. B. Clarke, and A. W. Bennett took part.—The Secretary then read a paper by Mr. H. N. Ridley, of Singapore, on two new genera of orchids from the East Indies.

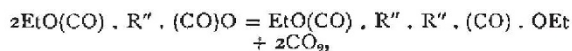
Zoological Society, April 7.—F. Du Cane Godman, F.R.S., Vice-President, in the chair.—The Secretary read a report on the additions that had been made to the Society's Menagerie during the month of March; and called special attention to a young example of the Ounce or Snow-Leopard (*Felis uncia*), new to the collection, and to a Small-clawed Otter (*Lutra leptonyx*) from India, being the second specimen of this Otter acquired by the Society; also to a specimen of a Lhuys' Impeyan (*Lophophorus lhuysi*) from Szechuen, Western China, obtained by Mr. A. G. Pratt, during his recent visit to that country, being the first example of the species that has reached Europe.—The Secretary exhibited the drawing of a female Antelope (*Tragelaphus gratus*), with a young one, now living in the Zoological Garden, Amsterdam, which had been obligingly sent to him by Heer C. Kerbert, the Director of that Garden.—The Secretary exhibited (on behalf of Mr. W. L. Selater, Deputy Superintendent of the Indian Museum, Calcutta) a specimen of a Duck, apparently a hybrid between the Mallard (*Anas boschas*) and the Gadwall (*A. strepera*), which had been lately obtained in the vicinity of Calcutta.—Mr. T. D. A. Cockerell read a paper on the geographical distribution of Slugs. The author divided the known Slugs into six families: Succineidae, Vaginulidae, Arionidae, Limacidae, Testacellidae, and Selenitidae, under which he grouped fifteen sub-families. The Janellidae were reduced to a sub-family of Succineidae, and the generic nomenclature of the whole group was revised, two new genera and one new sub-genus being named. The Philomycidae were made a sub-family of the Arionidae. The distribution of each sub-family, and of all recognizable genera, was discussed in some detail. Under the *Veronicellinae* a new sub-genus *Imerinia*, from Madagascar, was indicated.—A communication was read from Dr. Alcock, Surgeon-Naturalist to H.M. Indian Survey steamer *Investigator*, containing a description of *Saccogaster maculatus*, a viviparous Bathybial Fish from the Bay of Bengal.—Prof. F. Jeffrey Bell read some observations on *Bathybiaster vexillifer*, a Star-fish originally described by Sir Wyville-Thomson, of which the typical specimen had lately been received by the British Museum.—Mr. G. A. Boulenger gave an account of the Siluroid fishes obtained by Dr. H. von Ihering and Herr Sebastian Wolff in the Province of Rio Grande do Sul, Brazil.—Mr. F. E. Beddard read a paper giving some account of the anatomy of the Patagonian Cavy (*Dotichotis patagonica*) from specimens recently living in the Society's Gardens.

Mathematical Society, April 9.—Major MacMahon, R.A., F.R.S., Vice-President, in the chair.—Major MacMahon (Mr. J. J. Walker, F.R.S., taking the chair *pro tem.*) read a paper on the analytical forms called trees, with applications to

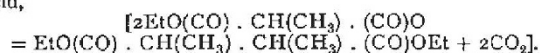
the combinations of certain electrical quantities and to the compositions of multipartite numbers. A discussion followed, in which Mr. Kempe, F.R.S., Mr. S. Roberts, F.R.S., and Mr. J. Hammond took part.—Mr. Kempe spoke on the flaw in his proof of the map-colour theorem which had been pointed out by Mr. P. J. Heawood (see Appendix to vol. xxi. of the Society's Proceedings), and stated that he was still unable to solve the problem fully.—Mr. Tucker (Hon. Sec.) communicated a paper by Mr. Culverwell on compounded solutions in the calculus of variations.

## EDINBURGH.

Royal Society, April 6.—The Hon. Lord Maclaren, Vice-President, in the chair.—Sir Douglas Maclagan read an obituary notice, by the Right Hon. Lord Moncrieff, of Prof. Campbell Swinton.—Prof. Crum Brown read a paper, written by himself and Dr. James Walker, on the synthesis of dibasic acids by means of electrolysis. In a previous paper on this subject the authors described the behaviour of the ethyl-potassium salts of normal dibasic acids on electrolysis. These they found to yield always the diethyl esters of normal acids of the same series. In the present paper, extending their investigation to acids with side chains, they give an account of the alkyl derivatives of succinic acid as obtained by the electrolysis of ethyl-potassium-methyl-malonate and ethyl-potassium ethyl-malonate. The esters formed according to the general equation,



are evidently always symmetrical, so that from methyl-malonic acid one might expect to obtain symmetrical dimethyl-succinic acid,



This dimethyl-succinic acid contains two similarly situated asymmetrical carbon atoms, and is thus, like tartaric acid, capable of existence in four isomeric forms—two optically active, and two optically inactive, one of these latter (corresponding to racemic acid) being a compound or mixture in equal proportions of the two opposite optically active acids. As the optically active forms are produced in equal proportions by any purely chemical process from inactive materials, the authors were justified in expecting to obtain, by electrolysis, a mixture of the esters of the two inactive symmetrical dimethyl-succinic acids. The synthesis was conducted in precisely the same manner as in the previous experiments. The authors succeeded in separating and purifying two acids—one, the less soluble, having the melting-point  $193^\circ \text{C}$ ., the other melting at  $120^\circ\text{--}121^\circ \text{C}$ . These acids on analysis proved to have the same composition—both corresponding to the formula  $\text{C}_6\text{H}_{10}\text{O}_4$ —the acid melting at  $193^\circ \text{C}$ . being apparently identical with the para-dimethyl-succinic acid of Bischoff (melting-point  $194^\circ \text{C}$ .), while the other seems to be identical with his anti-s-dimethyl-succinic acid. This conclusion was further confirmed by measurements of the electrolytic conductivity of solutions of the acids. In a similar manner the authors performed the electrolysis of ethyl-potassium ethyl-malonate. As before, they obtained, in the pure state, two acids, one melting at  $192^\circ \text{C}$ . with decomposition, the other at  $130^\circ \text{C}$ . Analysis showed that these acids have the composition of diethyl-succinic acid, and, from their mode of formation, are symmetrical. These are evidently identical with the para-diethyl-succinic acid (melting-point  $192^\circ \text{C}$ ., with decomposition), and the anti-s-diethyl-succinic acid (melting-point  $129^\circ \text{C}$ .) of Bischoff and Hjelt. This conclusion also was further confirmed by measurements of electrolytic conductivity.—Prof. Tait read a paper on the Virial, with special reference to the isothermals of carbonic acid. He showed that the usual mode of writing the equation, with  $\beta(v - \beta)$  as the left-hand member (where the term  $\beta\beta$  is part of the virial), is incorrect except in the absence of molecular force. The true form of the (approximate) virial equation is

$$\beta = \bar{\beta} \left( 1 - \frac{(v - v^0)^2}{v(v + a)(v + \gamma)} \right) + \left( R + \frac{e - a}{v + a} \right) \frac{t - t^0}{v}$$

where  $\bar{\beta}$ ,  $v^0$ ,  $t$  belong to the critical point. A first rough estimate, based on Amagat's recent work, gives for carbonic acid the values  $\bar{\beta} = 72.6 \text{ atm.}$ ,  $v^0 = 0.0046$ . Andrews long ago care-

fully determined  $t = 30^{\circ}9$  C. The approximate values of the other constants are  $\alpha = 0.001$ ,  $\beta = 0.0008$ ,  $R = 0.00371$ , and  $\epsilon = 0.000021$ . With these, the above formula gives fair representations of the isothermals from  $0^{\circ}$  C. to  $100^{\circ}$  C., for ranges of pressure from 1 to 500 atm. These are, however, to be regarded as provisional values only. Further numerical work is required to determine them more exactly. The formula above is based on the assumptions (1) that the particles are hard spheres, and (2) that the absolute temperature is measured by the average energy of a free particle; and its agreement with experiment is regarded as strong evidence in favour of the truth of the second of these assumptions; which, in its turn, throws strong light upon the nature of the liquid, and even of the solid, state.

PARIS.

Academy of Sciences, April 6.—M. Duchartre in the chair.—On a system of equations from partial derivatives, by M. Émile Picard.—Transformation *in vitro* of lymphatic cells into *clasmatocytes*, by M. L. Ranvier. It is shown that the lymphatic cells of the frog may be transformed into ramified, immobile cells—that is, into *clasmatocytes*—by making a preparation of the peritoneal lymph and keeping it in a glass cell at a temperature of  $25^{\circ}$  C. for one hour.—On vaccination by minimum doses of vaccinating matter, by M. Ch. Bouchard. The results of numerous experiments indicate that vaccinating matters act efficaciously when the amount employed is only a small fraction of a milligram. In one experiment complete immunity was obtained by the total injection of 0.026 c.c. of the culture per kilogram of the subject.—Interpretation of the fire-ball painted by Raphael in his picture the “Madonna di Foligno,” by M. Daubrée (see NATURE, vol. xliii. p. 500, and *American Journal of Science*, March 1891).—The law according to which the sum of the distances from the moon to two certain stars varies in the function of time, by M. L. Cruls.—New nebulae discovered at Paris Observatory, by M. G. Bigourdan. This is a continuation of lists previously given, and contains a description of fifty-five new objects situated between nine and sixteen hours of right ascension.—Observations of the asteroid (308) discovered at Marseilles Observatory with the Eichens equatorial, by M. Borrelly. Observations for position were made on March 31 and April 1 and 4.—On the theory of surfaces applicable to a given surface, by M. J. Weingarten.—On the theory of applicable surfaces, by M. E. Goursat.—On an analytical problem which is connected with dynamical equations, by M. R. Liouville.—On regular continuous fractions relative to  $e^2$ , by M. H. Padé.—On the mode of vibration of membranes, and the rôle of the thyro-arytenoidean muscle, by M. A. Hubert.—Preparation and properties of iodide of boron, by M. Henri Moissan. (See Notes, p. 568.)—On a new compound of molybdenum, by M. E. Péchard. A description of permolybdic acid,  $Mo_3O_7$ , is given.—On a new method for the separation of iron from cobalt and nickel, by M. G. A. Le Roy. An electrolytic method is proposed for the separation of iron from cobalt and nickel.—On the asymmetry and the production of the rotatory power in the chlorides of the compound ammoniums, by M. J. A. Le Bel. The author shows that when four alcoholic radicals are substituted for the hydrogen in ammonium chloride, the molecule appears to attain an invariable geometrical form. This is experimentally proved by the existence of isomers and the appearance of rotatory power when these four radicals are different.—On the nitro-derivatives of dimethyl-ortho-anisidine, by MM. E. Grimaux and L. Lefevre.—On the transformation by heat from campho-sulpho-phenols to homologues of ordinary phenol, by M. P. Cazeneuve.—On terebenthene, by M. Raoul Varet.—On ethyl malonate, and ethyl-potassium malonate, by M. G. Massol.—On the micro-organisms found on grapes, and on their development during fermentation, by MM. V. Martinand and M. Rietsch.—Contribution to the study of the bleaching effect of the air, by MM. A. and P. Buisine.—Law of position of nervous centres, by M. Alexis Julien.—New observations on the oceanic sardine, by M. G. Pouchet.—On the supposed crane of Moctezuma II., by M. E. T. Hamy.—On the existence of tufts of andesite in the flysch of La Clusaz (Upper Savoy), by M. P. Termier.—On the phenomena consecutive to the alteration of the pancreas determined experimentally by an injection of paraffin in the *Wirsung* canal, by M. E. Hédon.—On the phenomena consecutive to the destruction of the pancreas, by

M. E. Gley.—Chemical researches on microbial secretions: transformation and elimination of the nitrogenous organic matter by the pyocyanic bacillus in a medium of a given culture, by MM. Arnaud and A. Charrin.

STOCKHOLM.

Royal Academy of Sciences, March 11.—On symbiosis as causing accessory secretions in the shells of marine Gastropoda, by Dr. Carl Aurivillius.—Researches on the amount of blood expelled from the heart, by Prof. Tigerstedt.—On pendulum observations made in the mines of Sala during 1890, by Prof. Rosén.—A report on a foreign tour undertaken to study constructions suitable for maritime purposes, by Herr Nystedt, marine engineer.—On the respiration of the Algæ, by Miss H. Lovén.—On the hydrography of the fiord of Gullmar, by Miss A. Palmqvist.—Observations mycologicæ; i. e. genere Russula, by L. Romell.—On the African species of the genus Xyris, by Dr. A. Nilsson.—An elementary demonstration of the fundamental proposition of the equation theory, by Dr. E. Phragmén.—On the cyclic system of Ribaucour, by Prof. Bäcklund.—The radiation of the clouds around the thermometric minima, by Dr. H. Hamberg.—Geological observations on Snaefellensand in the environs of the Faxebay in Iceland, by Dr. Th. Thoroddsen.—Derivation of a formula within the mathematical statistic, by Dr. G. Eneström.—Observations on the ephippia or the hivernal egg-capsules of *Daphnia pulex*, by Baron G. C. Cederström.

BOOKS, PAMPHLETS, and SERIALS RECEIVED.

A Dictionary of Applied Chemistry: Prof. T. E. Thorpe; vol. ii. (Longmans).—The Missouri Botanical Garden (St. Louis).—A Treatise on Plane Trigonometry: E. W. Hobson (Cambridge University Press).—Elementary Lessons in Heat, Light, and Sound: Prof. D. E. Jones (Macmillan and Co.).—The “Progressive” Euclid, Books 1 and 2: A. T. Richardson (Macmillan and Co.).—Magnetic Observations at the U.S. Naval Observatory, 1888 and 1889: J. A. Hoogewerff (Washington).—The Elements of Statics and Dynamics; Part 2, Elements of Dynamics: S. L. Loney (Cambridge University Press).—Traité Élémentaire d'Électricité: J. Foubert; 2me. édition (Paris, G. Masson).—The London and Middlesex Note-book (E. Stock).—Elementary Chemistry: W. J. Harrison (Blackie).—Familiar Objects of Every-day Life: J. Hassell (Blackie).

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