In order to weigh any number of pounds from 1 to $n$ inclusive, we have to factorize the expression

$$x^{-n} + x^{(-n-1)} + x^{-(n-2)} + \ldots + n^{-1} + 1 + x + \ldots + x^{n-2} + x^{n-1} + x^n,$$

which may be thrown into the form

$$\frac{1 - x^{2n+1}}{x^n (1 - x)}.$$

The solutions depend upon the composite character of the number $2n + 1$. There always exists the trivial solution consisting of $n$ ones, and when $2n + 1$ is prime, this constitutes the only solution.

Supposing $2n + 1$ the product of two primes, viz.

$$2n + 1 = st,$$

we may write

$$\frac{1 - x^{2n+1}}{x^n (1 - x)} = \frac{1 - x^{st}}{x^n (1 - x)} = \frac{1}{x^n} \cdot \frac{1 - x^s}{1 - x} \cdot \frac{1 - x^{st}}{1 - x^s}$$

$$= \frac{1 - x^s}{x^{\frac{s-1}{2}} (1 - x)} \cdot \frac{1 - x^{st}}{x^{\frac{s(t-1)}{2}} (1 - x^s)}$$

$$= \left( x^{-\frac{s-1}{2}} + x^{-\frac{s-3}{2}} + \ldots + x^{-1} + 1 + x + \ldots + x^{\frac{s-3}{2}} + x^{\frac{s-1}{2}} \right)$$

$$\times \left( x^{-\frac{st-s}{2}} + x^{-\frac{st-3s}{2}} + \ldots + x^{-s} + 1 + x^s + \ldots + x^{\frac{st-3s}{2}} + x^{\frac{st-s}{2}} \right).$$

The factors appearing in this last expression are of the required form, and the factorization indicates that any number from 1 to $n$ inclusive may be composed by means of $\frac{1}{2}(s - 1)$ ones, and $\frac{1}{2}(t - 1)$ $s$'s, if all the ones that are taken be taken *either* positively or negatively, and all the $s$'s also *either* positively or negatively.

The solution may be represented, according to the method before explained, by

$$1^{\frac{1}{2}(s-1)} \; s^{\frac{1}{2}(t-1)},$$

and the complete system of solutions will in the present case be denoted by

$$\begin{cases} 1^{\frac{1}{2}(st-1)}, \\ 1^{\frac{1}{2}(s-1)} \; s^{\frac{1}{2}(t-1)}, \\ 1^{\frac{1}{2}(t-1)} \; t^{\frac{1}{2}(s-1)}. \end{cases}$$

From the above, it is clear that there is a one-to-one correspondence between the solutions of the second class problem in regard to the number $n$ and the solutions of the first class problem in regard to the number $2n$. The theory of the second class problems is thus included in that of the first class problems. If we take the case considered in this journal of $n = 40$ and both scale-pans, the number of solutions will be the same as that for $n = 80$ and one scale-pan; the number depends upon the composite character of the integer 81, which is $3^4$; hence the number of solutions (see *ante*) is [4], which is $2^3$ or 8.

Corresponding to the several identities—

$$\frac{1 - x^{81}}{x^{40} (1 - x)} = x^{-40} + x^{-39} + \ldots + x^{-1} + 1 + x + \ldots + x^{39} + x^{40};$$

$$\frac{1 - x^{81}}{x^{39} (1 - x^3)} \cdot x (1 - x) = (x^{-39} + x^{-36} + \ldots + x^{-3} + 1 + x^3 + \ldots + x^{36} + x^{39}) (x^{-1} + 1 + x);$$

$$\frac{1 - x^{81}}{x^{36} (1 - x^9)} \cdot \frac{1 - x^9}{x^4 (1 - x)} = (x^{-36} + x^{-27} + \ldots + 1 + \ldots + x^{27} + x^{36}) (x^{-4} + x^{-3} + \ldots + 1 + \ldots + x^3 + x^4);$$

$$\frac{1 - x^{81}}{x^{36} (1 - x^9)} \cdot \frac{1 - x^9}{x^3 (1 - x^3)} \cdot \frac{1 - x^3}{x (1 - x)} = (x^{-36} + x^{-27} + \ldots + x^{36}) (x^{-3} + 1 + x^3) (x^{-1} + 1 + x);$$

$$\frac{1 - x^{81}}{x^{27} (1 - x^{27})} \cdot \frac{1 - x^{27}}{x^{13} (1 - x)} = (x^{-27} + 1 + x^{27}) (x^{-13} + x^{-12} + \ldots + x^{13});$$

$$\frac{1 - x^{81}}{x^{27} (1 - x^{27})} \cdot \frac{1 - x^{27}}{x^{12} (1 - x^3)} \cdot \frac{1 - x^3}{x (1 - x)} = (x^{-27} + 1 + x^{27}) (x^{-12} + x^{-9} + \ldots + x^{12}) (x^{-1} + 1 + x);$$

$$\frac{1 - x^{81}}{x^{27} (1 - x^{27})} \cdot \frac{1 - x^{27}}{x^9 (1 - x^9)} \cdot \frac{1 - x^9}{x^4 (1 - x)} = (x^{-27} + 1 + x^{27}) (x^{-9} + 1 + x^9) (x^{-4} + x^{-3} + \ldots + x^4);$$

$$\frac{1 - x^{81}}{x^{27} (1 - x^{27})} \cdot \frac{1 - x^{27}}{x^9 (1 - x^9)} \cdot \frac{1 - x^9}{x^3 (1 - x^3)} \cdot \frac{1 - x^3}{x (1 - x)} = (x^{-27} + 1 + x^{27}) (x^{-9} + 1 + x^9) (x^{-3} + 1 + x^3) (x^{-1} + 1 + x)$$

These are the eight solutions represented by

$$\begin{array}{l} 1^{40} \\ 1 \cdot 3^{13} \\ 1^4 \cdot 9^4 \\ 1 \cdot 3 \cdot 9^4 \\ 1^{13} \cdot 27 \\ 1 \cdot 3^4 \cdot 27 \\ 1^4 \cdot 9 \cdot 27 \\ 1 \cdot 3 \cdot 9 \cdot 27. \end{array}$$

The subject is more fully entered into in a paper by myself in the *Quarterly Journal of Pure and Applied Mathematics* for 1886.

P. A. MACMAHON.

## *THE SCIENTIFIC RESULTS OF THE OCCUPATION OF BRITISH NEW GUINEA.*

A VOLUMINOUS and extremely interesting report on the first year of the administration of British New Guinea by Sir William MacGregor was issued some time ago by the Colonial Office. It deals with the period ending June 30, 1889. One of the sections of the report deals with "scientific results," which we are glad to notice have a place like "finance," "legislation," "trade and shipping," and the other usual divisions of these colonial reports. In sending the report home, Sir Henry Norman, the Governor of Queensland, observes that it is fortunate that the administrator is most anxious to obtain the best scientific results on his visits and tours, and that he is well able to judge for himself in such matters. The scientific collections, therefore, are made with judgment, and the various reports on collections are of interest and value. Sir William MacGregor himself, in summing up the scientific results, says that during the year some addition was made to our knowledge of the natural history of the country. Unfortunately, it is not possible to set out fully the progress made, as the report on specimens sent to England had not reached him at the moment of writing. It is his hope, however, that in future all specimens collected may be examined in Australia, so that the information gained can be kept together and be summarized in each annual Report.

*Geology.*—Thirty-one small bags of specimens were examined by Mr. Jack, Government Geologist of Queensland. All except two were from the Louisiade and D'Entrecasteaux groups. Mr. Jack's report, which will be found to be interesting and valuable, is given in an appendix. A set of specimens, covering the route from Manu-Manu to the summit of the Owen Stanley Range, was submitted to careful examination by Mr. Rands, Assistant Government Geologist of Queensland, who at the same time classified the specimens collected in the Rigo district. Mr. Rand's report will also be found in an appendix. Although not forming any part of the work of the year, there is added to the same appendix a report on certain geological specimens collected by Mr. C. S. Wilkinson. These three reports practically contain all that is really known of the geology of the country.

*Ornithology.*—The greater portion of the birds obtained were classified by Mr. de Vis. His report, prepared after much careful labour, is added as an appendix. From it may be inferred that the probability is that no great addition will be made now to the more beautiful and gorgeous birds of British New Guinea.

*Reptiles.*—The reptiles collected during the latter part of the year were classified by Mr. de Vis, Director of the Queensland Museum. They include ten species of snakes. Mr. de Vis has reported that of these ten species only two, a death-adder and a whip-snake, are to be dreaded. A small batrachian, mentioned Mr. de Vis, is interesting as having been brought from the top of Mount Victoria. A note furnished by Mr. W. H. Miskin, giving the results of his examination of a collection of Lepidoptera, and a description by Mr. C. Hedley of a new Rhylida from the highest summit of the Owen Stanley Range, are also given in an appendix.

*Botany.*—All botanical specimens were sent to Baron Mueller. He has found time to classify the specimens forwarded to him, and his report is also included in the "White-Book." In this branch much yet remains to be done, the collecting of plants being attended with great difficulties in New Guinea. It thus appears that the scientific work is contained in a series of appendices by specialists, in the shape of reports made on the various collections of Sir William MacGregor and his officers. Appendix B is a report by Prof. Liversidge on the hot springs of Ferguson Island in the D'Entrecasteau group; while D is a very long report from Sir William MacGregor himself on a tour made by him from Manu-Manu on the coast to the Owen Stanley Range in the interior. It gives *inter alia* a fascinating description of the mountain scenery in that great range. Appendix F contains the reports of Messrs. Jack and Rands on the geological collections. Mr. Jack observes that the specimens tell no connected tale such as would enable one to construct even a theory regarding the geology of the islands. They show, however, that palæozoic rocks, such as are the matrices of gold and other metallic deposits in Australia and elsewhere, are abundant, and that basaltic lavas are of common occurrence. The limestones may yield fossils which would be of great service in unravelling the structure of the islands. Mr. Rands, to whom were submitted the specimens collected by Sir William MacGregor on his expedition up the Vanapa river to the top of Mount Victoria, says that they enable one to form an idea of the geological character of the country drained by the Vanapa. The whole region consists almost entirely of schists, which become more highly metamorphosed as the loftier heights of the Musgrave Range and Mount Victoria are reached. On Mount Victoria the schists are very numerous, highly crystalline, and closely approaching to gneiss; on passing down the river, the country consists of clay, schists, and slates; while from near the mouth there are specimens of a but slightly altered sandstone. The report of Mr. de Vis, of the Brisbane Museum, on the birds is contained in Appendix G. The collection contained 161 specimens, representing 82 species, of which 13 appear to be hitherto unrecorded; "and of the apparent novelties one at least lays claim to generic rank." This is a very distinct kind of bower bird, obtained on Mount Knutsford, at an elevation of 11,000 feet, and rivalling the Regent Bird in beauty. "The name *Cnemophilus* (Mountain-slope Lover) has been appropriated to it, and the species I propose, with permission, to dedicate to yourself [Sir William MacGregor]. A second new bower bird, constituting a third species of the genus *Amblyornis*, and distinguished by a very ornate crest, will, if allowed, be honoured with the name of Lady MacGregor. It is well to note that the diversity in the structure of the bowers of this and of the other crested species of *Amblyornis* is far greater than the differences in their personal attributes. At your request the name of Mr. Belford, one of your party, has been associated with a capture in which he was concerned, a new honeyeater, of the genus *Melirrhophetes*. A similar compliment has been paid to another member of your collecting staff, Mr. C. Kowald, in connection with the beautiful genus of flycatchers, *Todopsis*. The number of species procured during the expedition to the Owen Stanley Range was 61, eight of them being apparently new to science. The expectations of ornithologists who have for some time been awaiting the exploration of that region will thus be in some measure fulfilled, notwithstanding that no new Birds of Paradise have been discovered. Perhaps, however, the greatest interest attaching to the ornithological results obtained arise from the fact that the decided change of climate observed at the altitude attained, over 13,000 feet, is not attended by a corresponding change in the types of bird life; it would seem that there is even here no infusion of forms characterizing temperate or cold latitudes. It is true that no birds were brought down from the highest points reached, but at 13,000 feet a flycatcher was procured which is essentially Australian in type. The presence of a blackbird, now first discovered in New Guinea, is not in this connection contradictory, since the genus *Merula* is represented in other of the Pacific Islands. Some interesting additions to our knowledge of the birds of the Louisiade Archipelago result from your visits to the islands within your jurisdiction. 21 species from East, Sudest, Ferguson, Rossel, and other isles have been determined; of these several cannot be identified with species previously known, as far as I am able to judge. As these birds were procured hurriedly, they doubtless represent but a very small proportion of the several faunas. If it were possible to station a collector on one of the larger islands, Sudest, for example, so that a fairly complete knowledge of its zoology could be obtained, science would be greatly benefited." The descriptive list of the birds which follow is very full and interesting. It is succeeded by a report by the same gentleman on the reptiles collected during the expedition; they consisted of two species of lizards, ten of snakes, and one frog. The snake-like lizard (*Lialis*) is common to Australia and New Guinea; the sleeping lizard (*Cyclodus*) is in Australia represented by other species; both are perfectly harmless. Of the snakes also, the greater majority are innocuous; the death-adder (*Acanthophis*) and the whip-snake (*Diemansia*) are, indeed, the only kinds to be dreaded. With the exception of the tree-snake (*Dendrophis*) the rest are the constrictors *Liasis*, *Chondropython*, *Aspidopython*, and *Enygrus*, and the colubine snakes *Lycodon*, *Mainophis*, and *Pappohis*. It is clear that deadly snakes are not to be added to the imaginary terrors of the Papuan climate. Four of the snakes represented are Australian species. The sole exampler of the batrachians is a little frog, which Mr. de Vis regards as a new species of its genus, *Michrohyla*. This is followed by Mr. Miskin's note on the collection of Lepidoptera. He says that a glance at the specimens, confirmed upon closer inspection, conveyed the impression that they represented only the fauna of the lower altitudes, although as a fact they were collected at some distance from the coast, proving, with but two or three exceptions, to be species already known to science. Sir William was unfortunately unable to give attention to this branch of zoology during his explorations of the higher altitudes, where new forms of the greatest interest might be anticipated to occur. He observes that the collection is interesting as showing the similarity of the New Guinea fauna with that of North East Australia, there being no less than 23 of the 53 species common to both regions; while of the 31 genera represented 25 are found in both countries.

Mr. Hedley in his note on the new Rhytida says that from the highest summit (13,000 feet) of the Owen Stanley Range two land shells were brought down by Sir W. MacGregor and committed for examination to the Queensland Museum. As these are the first traces of molluscan life collected in the New Guinea mountains, and as no form at all resembling them is described by Sig. Tapparone-Canefri, he feels some confidence in deciding them to be a hitherto unknown species. More globose than other of the genus, the glossy exterior, nacreous interior, characteristic colour and sculpture, stamp it as a Rhytida; and thus another genus is added to those common to Australia and New Guinea. Though well preserved both the specimens that furnish the following description are "dead shells"; one is slightly darker and less globose than its fellow:—*Rhytida Globosa*, n.s. Shell, depressed-globose, thin, translucent, perforate, very glossy; whorls $4\frac{1}{2}$, the earlier flattened, the latter rounded, rather rapidly increasing, the last a little expanded, not descending at the aperture; colour, reddish-chestnut above, lighter beneath, first three whorls bleached nearly white; sculpture almost effaced on the body whorl, where nearly obsolete spiral impressed lines cross the faint irregular growth lines; the earlier whorls exhibit fine close oblique striae cut by fine spiral grooves, a pitted (not striated) surface is offered by the first whorl and a half, which seem embryonic; suture impressed, slightly crenulated, bordered beneath by a narrow white band, which is in turn edged by a black line; aperture ovate, oblique; peristone simple above, slightly reflected below; interior bluish-white, probably iridescent when fresh; columella wall overlaid by a thin deposit; umbilicus narrow, partially hidden by the reflected peristone at its junction with the base; base a little inflated. Greater diam. 17 mm., lesser 14 mm., height 10 mm. A report from Baron von Mueller on the Papuan Highland plants collected brings the scientific part of the "White-Book" to a conclusion. This is somewhat lengthy to reproduce in full now, and it is not easy to summarize it adequately. Readers must therefore be referred to the volume itself, where the report appears at pp. 125–129.