Sailing Flight of Large Birds over Land.

THE explanation of this, in NATURE (p. 518), seems inadequate.

If a large body of air be moving uniformly both in respect of direction and of velocity, no matter at what rate, it might as well be perfectly motionless, in respect of its ability to aid the flight of a bird that is simply floating in it. But in fact the air never is motionless: it moves with the earth, from west to east, at the rate, let us assume, of 500 miles an hour. To a bird floating in the air, whether the earth beneath it moves exactly as the air moves, or not, must be a matter of perfect indifference. The earth's relative motion does not affect it. I must myself adhere to the explanation which I gave in a former number of NATURE (vol. xxviii. p. 28), that the birds avail themselves of differences of the movement of the air, in respect of velocity, or of direction, or of both. Mr. S. E. Peal has noticed that "flocks which drift over the hills recover their position on the plains by descending to windward." This is simple enough. The wind is flowing from the plains towards the hills. It rises then as it flows, and has many inequalities in its direction and rate. On entering a gorge a narrow current of air would be thrown upwards with very rapid ascent. Of all the inequalities the birds know how to avail themselves. R. COURTENAY.

Tean Vicarage, September 28.

A Remarkable Meteor.

ON Sunday, September 29, at 7.30 p.m., I observed a very brilliant meteor falling nearly perpendicular a little to the west of north. Its progress towards the earth appeared to be much slower than is usually the case with such bodies, the heavens being illuminated for several seconds. The meteor was of a bright sapphire hue; preceding it were a few drops of bright fiery red, whilst following it came a brilliant trail of light. It seems to have been pretty generally observed throughout Ireland, and letters to the Press from counties Roscommon, Galway, Kilkenny, and Kildare, testify to the interest it has awakened in the country. RICHARD CLARK.

113 Upper Leeson Street, Dublin, October 7.

THE METHOD OF QUARTER SQUARES.[1]

THE method of quarter squares consists in the use of the formula

$$ab = \tfrac{1}{4}(a+b)^2 - \tfrac{1}{4}(a-b)^2$$

to effect the multiplication of two numbers, a and b. If we are provided with a table giving the values of $\frac{1}{4}n^2$ up to a given value of n, we may obtain, by the aid of this formula, without performing any multiplication, the product of any two numbers whose sum does not exceed the limit of the table.

The method is specially interesting on account of the great simplicity of the formula, by means of which a table of double entry may be replaced by one of single entry. How great a transformation is effected by such a change is evident, if we consider that the largest existing multiplication table of double entry reaches only to 1000 × 1000, and forms a closely-printed folio of 900 pages, but that a table of quarter squares of the same extent (*i.e.* of $\frac{1}{4}n^2$ up to $n = 2000$) need only occupy 4 octavo pages. The disparity becomes even more conspicuous as the limit of the table is extended, for a table of double entry extending to 10,000 × 10,000, would require nearly 100 folio volumes; and one extending to 100,000 × 100,000, would require nearly 10,000 volumes; whereas the corresponding quarter-square tables need only occupy 40 and 400 octavo pages respectively.

The use of a table of squares in effecting multiplications was recognized as far back as 1690, when Ludolff published his large table of squares, extending to 100,000. In the introduction to the table Ludolff explained how it could be employed in multiplications. In order to multiply a and b the table is to be entered with $a+b$ and $a-b$ as arguments, and the difference of the corresponding squares divided by 4. If a and b are both even, or both uneven, their sum and difference will both be even numbers, so that $\frac{1}{2}(a+b)$ and $\frac{1}{2}(a-b)$ will be integers. In either of these two cases we may therefore enter the table with the semi-sum and semi-difference of the numbers as arguments, the product being the simple difference of the corresponding squares.

It was not, however, till 1817 that a table of *quarter* squares (*i.e.* of $\frac{1}{4}n^2$ for argument n) was published, for the purpose of facilitating multiplications. If n be uneven, $\frac{1}{4}n^2$ consists of an integer and the fraction $\frac{1}{4}$. This fraction $\frac{1}{4}$ may be ignored in the use of the table, for if either $a+b$ or $a-b$ is uneven, the other is so too; the fraction $\frac{1}{4}$ therefore occurs in both squares, and disappears from their difference. It may therefore be omitted from the table.

The table of 1817, which contained the first practical application of the method, was published by Antoine Voisin, at Paris, under the title "Tables des Multiplications; ou, Logarithmes des Nombres entiers depuis 1 jusqu'à 20,000." It is curious that Voisin should have called a quarter square a logarithm: he called a the root, and $\frac{1}{4}a^2$ its logarithm. His table extended to 20,000, and was thus available for multiplications up to 10,000 × 10,000. On the title-page Voisin described it as effecting multiplications up to 20,000 by 20,000. This statement is justified by the formula

$$ab = 2\{\tfrac{1}{4}a^2 + \tfrac{1}{4}b^2 - \tfrac{1}{4}(a-b)^2\},$$

by which the product was to be obtained if the sum of the numbers exceeded 20,000, the method of quarter squares being then no longer available. It is to be observed, however, that this formula requires three entries besides the final duplication.

Almost simultaneously (1817) a similar table, of the same extent, was published independently by A. P. Bürger at Carlsruhe. The method was rediscovered by J. J. Centnerschwer, who published a table of the same extent in 1825 at Berlin. In 1832, J. M. Merpaut published, at Vannes, a table of quarter squares extending to 40,000. In 1852, Kulik (well known for his large table of squares and cubes to 100,000), who had again rediscovered the method, published a table extending to 30,000. In 1856, Mr. S. L. Laundy published, at London, the largest table of quarter squares which had appeared previous to the publication of the present table. Laundy's table extends to 100,000. It was intended that the multiplications should be effected by means of quarter squares if the sum of the numbers did not exceed 100,000, but other five-figure numbers were to be multiplied by means of Voisin's three-entry formula referred to above.

It is this change of method that has detracted so greatly from the value of Laundy's fine table. It is evident that the table should have been carried to double its actual extent, *i.e.* to 200,000, so that any two five-figure numbers could be multiplied together by means of the two-entry formula. The late General Shortrede constructed such a table, but it was never printed. In the work under notice Mr. Blater carries the table as far as 200,000; so that, more than sixty years after the publication of the first table effecting the multiplication of two four-figure numbers, the extension to five figures has at last been completed.

The method of quarter squares has had no opportunity of a fair trial in the absence of a table extending to 200,000. Considering the many purposes to which Crelle's tables (which give the product of any two three-figure numbers by a single entry) are continually applied, it is perhaps surprising that no general use should ever have been made of a table which in a very small compass gives, by only two entries, the product of two four-figure numbers. Still it is clear that the full power of the method

[1] "Table of Quarter Squares of all Whole Numbers from 1 to 200,000 for simplifying Multiplication, Squaring, and Extraction of the Square Root, and to render the Results of these Operations more certain." Calculated and published by Joseph Blater. (London: Trübner and Co., 1888.)