Planet.	Rises.	Souths.	Sets.	Right asc. and declination on meridian.	
	h. m.	h. m.	h. m.	h. m.	° ′
Mercury..	3 6	10 54	18 42	6 56·0	19 10 N.
Venus.....	4 23	12 20	20 17	8 22·3	20 34 N.
Mars......	12 49	17 48	22 47	13 51·2	12 34 S.
Jupiter....	15 10	19 34	23 58	15 37·6	18 37 S.
Saturn....	4 58	12 43	20 28	8 45·4	18 43 N.
Uranus...	11 8	16 47	22 26	12 51·0	4 46 S.
Neptune..	0 11	7 58	15 45	4 0·2	18 55 N.

* Indicates that the setting is that of the following morning.

Comet Sawerthal.

July.	h.	Right Ascension. h. m.	Declination. ° ′
22	0	1 7·9	51 59 N.
26	0	1 7·3	52 38

Occultations of Stars by the Moon (visible at Greenwich).

July.	Star.	Mag.	Disap.	Reap.	Corresponding angles from vertex to right for inverted image.
			h. m.	h. m.	° °
22	o Sagittarii	4	0 42	1 15	173 235
23	20 Capricorni	6	21 35	22 46	64 269
26	74 Aquarii	6	0 57	2 11	80 308

July.	h.	
23	—	Total eclipse of Moon: first contact with penumbra 2h. 57m.: first contact with shadow 3h. 55m., shortly after which, at 4h. 10m., the Moon sets at Greenwich.
23	22	Jupiter stationary.
24	4	Venus at least distance from the Sun.
27	13	Venus in conjunction with and 0° 35′ north of Saturn.

Variable Stars.

Star.	R.A.	Decl.			
	h. m.	° ′		h. m	
U Cephei	0 52·4	81 16 N.	July 25,	20 50	*m*
R Piscium	1 24·9	2 18 N.	,, 22,		*M*
W Virginis	13 20·3	2 48 S.	,, 25,	3 0	*M*
U Boötis	14 49·2	18 9 N.	,, 22,		*m*
δ Libræ	14 55·0	8 4 S.	,, 27,	0 18	*m*
V Coronæ	15 54·5	39 55 N.	,, 26,		*M*
U Ophiuchi	17 10·9	1 20 N.	,, 24,	2 50	*m*
			,, 24,	22 58	*m*
U Sagittarii	18 25·3	19 12 S.	,, 27,	1 0	*m*
β Lyræ	18 46·0	33 14 N.	,, 27,	23 0	*M*
η Aquilæ	19 46·8	0 43 N.	,, 25,	23 0	*M*
X Cygni	20 39·0	35 11 N.	,, 25,	0 0	*M*
δ Cephei	22 25·0	57 51 N.	,, 23,	2 0	*m*
R Pegasi	23 1·0	9 56 N.	,, 28,		*M*

M signifies maximum; *m* minimum.

Meteor-Showers.

	R.A.	Decl.	
Near δ Cassiopeiæ	20°	59° N.	Very swift. Streaks.
The *Perseids*	25	53 N.	Swift. Streaks.
The *Aquarids*	340	13 S.	Max. July 28.

GEOGRAPHICAL NOTES.

THE last survey of the Austrian Alps, we learn from the Proc. R.G.S., has already led to some important, if not altogether unexpected results. Thus the Marmolata, the highest dolomite, is reduced from 11,464 feet to 11,016 feet. The Antelao comes next, reaching, according to the new Italian survey, 10,874 feet. Mr. D. Freshfield pointed out in 1875, in his "Italian Alps," that the two highest points of the Primiero group do not differ by 159 metres, as then indicated in the Government survey, but are almost equal in height. The new measurements show a difference of only 16 feet between them, and reverse the advantage. The figures are subjoined:—

	Last survey.	Old survey.	Previous Cadaster measurement.
	m.	m.	m.
Cima di Vezzana	3191	3061	3317
Cimon della Pala	3186	3220	3343

The Cima di Vezzana is therefore 10,470 feet and the Cimon della Pala 10,454 feet. The remaining peaks of the Primiero group gain or lose only a few feet by the new measurements.

MR. W. J. ARCHER, British Vice-Consul at Chiengmai, has written an interesting Report of a journey he made in his district last year. This journey extended north along the Meping River, north-east to Chiengsin on the Cambodia River, south and east to Nan on the Nam Nan, then westwards across the Meyom, by Lakhon to Chiengmai. Several maps accompany the Report, which add considerably to our knowledge of the topography of the region visited. Mr. Archer, writing of the new capital of Müang Fāng, describes the manner in which this and similar new settlements were formed in Siam. In such new colonies, as the people spread out over the districts around, other settlements were gradually formed at a distance from the capital. A large body of immigrants, or a number of families from the same locality, generally form a separate settlement, especially if they are of a different race from the original settlers; and if they settle in the capital they usually have a separate quarter allotted to themselves. This is characteristic of all the settlements in Siam, both in the larger cities and in the provinces. In Bangkok the inhabitants of the different quarters have gradually become amalgamated; but not far from the capital the colonies of former captives of war still retain their language and customs, and keep up little intercourse with their conquerors. In the northern country the separation is as complete, and the area of Chiengmai, for example, is divided into numerous quarters, each inhabited almost exclusively by people of a different race; and many of the villages in the provinces are also colonies of refugees or captives. Mr. Archer is of opinion that the country of the Thai Yai (literally "great Siamese"), or its vicinity, is the cradle of the Thai people, who thence gradually flowed southward. The Thai family has numerous divisions, differing more or less in appearance, language, and costume, though it is not difficult to trace the common type through all. The whole subject of the gradual development and modifications of the Thai race is a very interesting one from an ethnological point of view, and, Mr. Archer thinks, well worthy of research for the light it may throw on the early history of Indo-China. Mr. Archer gives many useful notes on the various hill-tribes of the country, whose distribution and characteristics deserve careful investigation. It is to be hoped he may have further opportunities of exploring the region and collecting additional information.

THE Council of the Russian Geographical Society have issued a memorandum with regard to the teaching of geography in the Universities. This memorandum will probably be taken as a basis for the impending organization of University teaching and degrees in geography in Russia. "Geography," the Council write, "being a study of the laws and associations of phenomena of the physical and organic life of the earth, it implies a serious preliminary study of natural sciences. Without a serious knowledge of the laws of physics, it is impossible to reason upon the laws dealing with the physical features of the globe. For recognizing its true place in the solar system, its figure and movements, the knowledge of astronomy and geodesy is absolutely necessary. The origin of the present features of the surface of the earth cannot be dealt with without a knowledge of geology and mineralogy. Botany and zoology are necessary for studying the laws of the distribution of organisms; while a knowledge of anatomy and physiology is necessary for the study of anthropology, phyto-geography, zoo-geography, and anthropo-geography, and so on." The experience of the German Universities having shown how difficult it is for the student to master all these subjects if he merely follows the usual lectures of the Natural Sciences Faculty, the Council express a hope that special courses, appropriate to the requirements of geographical students, may be opened in physics, astronomy and geodesy, chemistry, mineralogy and petrography, geology and the study of soils (a branch which has lately received a good deal of attention in Russia), zoology, anatomy and zootomy, physiology, history, literature, comparative philology, and the leading principles of political economy and statistics. Psychology being intrusted in Russian Universities only to Professors chosen from among the clergy, the Council urge that it should be introduced into the Natural History Faculty. As to geography proper, they advise, first, that there shall be two separate Professors for geography and anthropology, and point out the absolute impossibility of combining both sciences in one professorship. They propose, moreover, to divide the course of geography into two distinct parts, physical geography (*Erdkunde*) and special geography (*Länderkunde*). Historical geography is excluded from the programme, its contents belonging partly to history and partly to the *Länderkunde*. Although fully recognizing the difficulty of having lectures in all the above-named subjects especially appropriated to the needs of geography, the Council suggest that *privat-docents* might supply the new want. But if this is found to be impossible, they advise that the students who wish to take either geography or anthropology as their specialty should be left to select in the above-named group of sciences those subjects which would best suit them. Students might thus take any one of the three chief directions opened to the geographer—namely, that of the geologist-geographer, the biologist-geographer, or the anthropologist-geographer.

THE MULTIPLICATION AND DIVISION OF CONCRETE QUANTITIES.[1]

I HAVE recently been laying stress on the fact that the fundamental equations of mechanics and physics express relations among *quantities*, and are independent of the mode of measurement of such quantities; much as one may say that two lengths are equal without inquiring whether they are going to be measured in feet or metres; and indeed, even though one may be measured in feet and the other in metres. Such a case is, of course, very simple, but in following out the idea, and applying it to other equations, we are led to the consideration of products and quotients of concrete quantities, and it is evident that there should be some general method of interpreting such products and quotients in a reasonable and simple manner. To indicate such a method is the object of the present paper.

For example, I want to justify the following definition, and its consequences: Average velocity is proportional to the distance travelled and inversely proportional to the time taken, and is measured by the distance divided by the time, or, in symbols, $v = s \div t$. As a consequence of this, the distance travelled is equal to the average velocity multiplied by the time, or $s = vt$. The following examples will serve to illustrate what I mean:—

(i.) If a man walks 16 miles in 4 hours, his average speed is $\frac{16 \text{ miles}}{4 \text{ hours}} = 4 \times \frac{1 \text{ mile}}{1 \text{ hour}} = 4$ miles an hour, the symbol $\frac{1 \text{ mile}}{1 \text{ hour}}$ denoting a speed of a mile an hour, in accordance with the definition.

Similarly, $\frac{1 \text{ foot}}{1 \text{ second}}$, or shortly, $\frac{\text{ft.}}{\text{sec.}}$, denotes a velocity of a foot per second. The convenience of this notation is that it enables us to represent velocities algebraically, and to change from one mode of measurement to another without destroying the *equation*.

$$\text{Thus } \frac{16 \text{ miles}}{4 \text{ hours}} = \frac{4 \text{ miles}}{1 \text{ hour}} = \frac{4 \times 1760 \times 3 \text{ feet}}{60 \times 60 \text{ seconds.}} = 5{\cdot}9 \frac{\text{ft.}}{\text{sec.}}$$
$= 5{\cdot}9$ feet per second.

(ii.) The distance travelled in 40 minutes by a person walking at the rate of $4\frac{1}{2}$ miles an hour $= \frac{4\frac{1}{2} \text{ miles}}{1 \text{ hour}} \times 40 \text{ minutes} = \frac{4\frac{1}{2} \text{ miles}}{3} \times 2 = 3$ miles.

Such concrete equations are used by a considerable number of people, I believe, but I have not seen any attempt at a general method of interpreting the concrete products and quotients involved.

Now, I think I cannot do better by way of clearing the ground before us than quote what Prof. Chrystal says in his "Algebra" about multiplication and division. He begins by saying that multiplication originally signified mere abbreviation of addition; and then (on p. 12) he says:—

"Even in arithmetic the operation of multiplication is extended to cases which cannot by any stretch of language be brought under the original definition, and it becomes important to inquire what is common to the different operations thus comprehended under one symbol. The answer to this question, which has at different times greatly perplexed inquirers into the first principles of algebra, is simply that what is common is the formal laws of operation [the associative, commutative, and distributive laws]. These alone define the fundamental operations of addition, multiplication, and division, and anything further that appears in any particular case is merely a matter of some interpretation, arithmetical or other, that is given to a symbolical result, demonstrably in accordance with the laws of symbolical operation."

"*Division*, for the purposes of algebra, is best defined as the inverse operation to multiplication."

I will begin by considering instances, and then go on to the general case.

A product of a number and a concrete quantity presents no difficulty. All that is necessary is to define that the order of stating the product shall not alter its meaning—that is, that the *commutative* law shall hold—that,

$$e.g., 2 \times 1 \text{ foot} = 1 \text{ foot} \times 2 = 2 \text{ feet.}$$

The *distributive* law is satisfied; thus,

$$2 \text{ feet} + 3 \text{ feet} = (2 + 3) \text{ feet} = 5 \text{ feet.}$$

In interpreting the meaning of the product of two concrete quantities, we have to be careful that in the interpretation nothing shall violate the laws of *numerical* multiplication; *i.e.* if any numerical factors occur, they must be able to be multiplied in the ordinary way, and placed before the final concrete product, which must, of course, represent something which varies directly with both quantities.

Thus 4 feet $\times$ 2 yards must be equal to 8×1 foot $\times$ 1 yard.

Now a *rectangle*, whose sides are 4 feet and 2 yards, is eight times the rectangle whose sides are 1 foot and 1 yard, so that, if we define the product of two lengths as representing a rectangle whose sides are these lengths respectively, we are not violating any multiplication law as regards the numerical multipliers; and we can compare one such rectangle with any other whose sides are of different lengths, by ordinary multiplication and division among such numbers as arise, and by interpretation of the concrete products in accordance with the definition.

$$\begin{aligned}\text{Thus, } 4 \text{ feet} \times 2 \text{ yards} &= 8 \times 1 \text{ foot} \times 1 \text{ yard,} \\ &= 24 \times 1 \text{ foot} \times 1 \text{ foot,} \\ &= 24 \text{ square feet,} \\ &= 24 \times 12 \text{ inches} \times 12 \text{ inches,} \\ &= 3456 \text{ square inches,} \\ &\quad \text{\&c.}\end{aligned}$$

Here we have applied the commutative law so as to bring the numerical factors together for multiplication, and have interpreted the remaining concrete products in accordance with the definition.

The general result is that $ab = \alpha\beta \,.\, a'b'$, if $a = \alpha a'$, and $b = \beta b'$, *i.e.* a rectangle whose sides are a, b is $\alpha\beta$ times a rectangle with sides a', b', if $a = \alpha a'$, and $b = \beta b'$.

From this example I think we can see that a concrete product may properly be used to represent any quantity that varies directly as the several concrete factors, and that, being so represented, it may, by use of the ordinary rules of multiplication, be compared with any other concrete product of the same kind; that is to say, that, *generally*, $ab = \alpha\beta \,.\, a'b'$, if $a = \alpha a'$, and $b = \beta b'$, where α, β are numerical factors, and a, a' are different amounts of one kind of quantity, and b, b' of another kind.

Similarly, a concrete quotient may be used to represent a quantity which varies directly as the concrete numerator and inversely as the concrete denominator, and may, by the ordinary rules of multiplication and division, be compared with any other quantity of the same kind.

Indeed, I may go further and assert that a concrete product or quotient (the latter including the former) MUST, if it is to have any meaning at all, represent a quantity varying directly as the concrete factors in the numerator and inversely as those in the denominator, and that the general use of such representation is for comparison of the complex quantity with a standard of the same kind. Or, generally, we may say it should be used, whenever we wish, in our work, to give as full and explicit a representation to the complex quantity as possible.

The operation of multiplying [and dividing] concretes may be separated into two parts: the *formation* of the products, and the *simplification* of them; and this latter process may be again considered in two parts: the simplification of the numerical factors, *i.e.* ordinary multiplication and division, and the simplification of the concrete factors, *i.e.* cancelling where possible, and, finally, interpretation.

[1] Paper read at the General Meeting of the Association for the Improvement of Geometrical Teaching, on January 14, 1888, by A. Lodge, Cooper's Hill, Staines.