July.	h.	
16 ...	13 ...	Mars in conjunction with and 6° 40′ south of the Moon.
18 ...	17 ...	Jupiter in conjunction with and 4° 5′ south of the Moon.
20 ...	0 ...	Mercury stationary.

Variable Stars.

Star.	R.A. h. m.	Decl. ° ′		h. m.
U Cephei	0 52·4 ...	81 16 N. ...	July 15,	21 31 *m*
			,, 20,	21 11 *m*
W Virginis	13 20·3 ...	2 48 S. ...	,, 16,	22 0 *m*
δ Libræ	14 55·0 ...	8 4 S. ...	,, 20,	0 44 *m*
U Coronæ	15 13·6 ...	32 3 N. ...	,, 15,	21 43 *m*
W Herculis	16 31·3 ...	37 34 N. ...	,, 20,	*m*
U Ophiuchi	17 10·9 ...	1 20 N. ...	,, 19,	2 4 *m*
			,, 19,	22 12 *m*
W Sagittarii ...	17 57·9 ...	29 35 S. ...	,, 16,	21 0 *m*
Z Sagittarii	18 14·8 ...	18 55 S. ...	,, 16,	0 0 *m*
T Serpentis	18 23·4 ...	6 14 N. ...	,, 20,	*M*
β Lyræ	18 46·0 ...	33 14 N. ...	,, 15,	1 0 *M*
R Lyræ	18 51·9 ...	43 48 N. ...	,, 18,	*m*
R Cygni	19 33·8 ...	49 57 N. ...	,, 19,	*M*
S Aquilæ	20 6·5 ...	15 17 N. ...	,, 21,	*m*
S Delphini	20 37·9 ...	16 41 N. ...	,, 18,	*m*
X Cygni	20 39·0 ...	35 11 N. ...	,, 21,	0 0 *m*

M signifies maximum; *m* minimum.

Meteor-Showers.

	R.A. °	Decl.	
The *Perseids*	23 ...	50 N. ...	Swift, streaks.
Near γ Draconis	269 ...	51 N. ...	Swift.
,, α Lacertæ	336 ...	49 N. ...	Swift, short.

GEOGRAPHICAL NOTES.

THE Geographical Society of Paris have decided to avail themselves of the Universal Exhibition at Paris, next year, by convening an International Congress of the Geographical Sciences, to meet in the month of August. There will be two classes of members, subscribing respectively 40 and 20 francs, and each member will be entitled to receive a copy of the publications of the Congress and have a vote in the questions discussed at the meetings. Each Society represented at the Congress will be invited to submit a report on the voyages, explorations, and publications which have most contributed, in the country to which it belongs, to the progress of geography during the past hundred years; the combined reports will afterwards be published with the names of their authors.

DR. H. MEYER has made some important corrections in the preliminary account of his ascent of Kilimanjaro. After verifying and correcting his barometrical observations, he admits that the previously accepted height of 18,700 feet is more accurate than that given by himself, 19,850 feet. He then refers to the dense mist which prevented him from seeing beyond a wall of inaccessible ice, 130 feet high, which his first account indicated as being the terminal point of the peak. It results from these observations that Dr. Meyer did not reach to within 820 feet of the summit of Kilimanjaro, which therefore still remains unconquered.

M. JULES BORELLI, the French traveller, who accompanied M. Rimbaud last year in his interesting journey from Antotto to Harar, is engaged in exploring the country to the south-west of Shoa. The Paris Geographical Society has received some of the results accruing from his journey from Antotto to Jiren, which is situated in 7°42′ N. latitude, and 34°35′ E. longitude. Among these results is the discovery of the sources of the River Hawash, which lie at the foot of Mount Ilfata at the extremity of the Meca range, and not near Mount Dandi, as hitherto supposed. On the summit of the latter peak the traveller found a double lake resembling in shape the figure 8, which is of considerable extent and depth; an affluent of the Gudar, and thus of the Abbay, issues from this lake. He also discovered a deep lake at the bottom of the immense crater mountain known as Mount Harro; the surroundings of this sheet of water are described by the traveller as of incomparable beauty. From this lake, which is named by the natives Wancit, a stream issues and joins the Walga, the source of the latter river being in the summit of Mount Harro. Dr. Traversi, the Italian explorer, made in June, 1887, an excursion into the mountainous region of Urbanagh, lying to the east of the district now being explored by M. Borelli. The chief result of this journey of Dr. Traversi is to throw light on the problem of the hydrographical systems of the Somali and Galla countries. From the summit of Mount Gafat he was able to comfirm his previous observations made near the Suai Lake, with reference to the three lakes abovementioned and their interconnection.

ON CERTAIN INEQUALITIES RELATING TO PRIME NUMBERS.

I SHALL begin with a method of proving that the number of prime numbers is infinite which is not new, but which it is worth while to recall as an introduction to a similar method, by series, which will subsequently be employed in order to prove that the number of primes of the form $4n + 3$, as also of the form $6n + 5$, is infinite.

It is obvious that the reciprocal of the product

$$\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\left(1 - \frac{1}{p_3}\right) \cdot \cdot \cdot \cdot \left(1 - \frac{1}{p_{N.p}}\right)$$

(where p_i means the ith in the natural succession of primes, and $p_{N.p}$ means the highest prime number not exceeding N)[1] will be equal to

$$\tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} + \tfrac{1}{6} + \cdot \cdot \cdot \cdot + \frac{1}{N} + R,$$

and therefore greater than log N (R consisting exclusively of positive terms).

Hence

$$\left(1 + \frac{1}{p_1}\right)\left(1 + \frac{1}{p_2}\right) \cdot \cdot \cdot \cdot \left(1 + \frac{1}{p_{N.p}}\right) > M \log N,$$

where

$$M = \left(1 - \frac{1}{p_1^2}\right)\left(1 - \frac{1}{p_2^2}\right) \cdot \cdot \cdot \cdot \left(1 - \frac{1}{p_{N.p}^2}\right),$$

and is therefore greater than $\frac{2}{\pi}$.

Hence the number of terms in the product must increase indefinitely with N.

By taking the logarithms of both sides we obtain the inequality

$$S_1 - \tfrac{1}{2}S_2 + \tfrac{1}{3}S_3 - \tfrac{1}{4}S_4 + \cdot \cdot \cdot \cdot > \log \log N,$$

where in general S_i means the sum of inverse ith powers of all the primes not exceeding N; and accordingly is finite, except when $i = 1$, for any value of N. We have therefore

$$S_1 > \log \log N + \text{Const.}$$

The actual value of S_1 is observed to differ only by a limited quantity from the second logarithm of N, but I am not aware whether this has ever been strictly proved.

Legendre has found that for large values of N

$$(1 - \tfrac{1}{3})(1 - \tfrac{1}{5}) \cdot \cdot \cdot \cdot \left(1 - \frac{1}{p_{N.p}}\right) = \frac{1 \cdot 104}{\log N}.$$

Consequently

$$\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \cdot \cdot \cdot \cdot \left(1 - \frac{1}{p_{N.p}}\right) = \frac{\cdot 552}{\log N}.$$

This would show that the value of our R bears a finite ratio to log N; calling it θ log N we obtain, according to Legendre's formula,

$$\frac{1}{1 + \theta} = \cdot 552, \text{ which gives } \theta = \cdot 811,$$

so that the nebulous matter, so to say, in the expansion of the reciprocal of the product of the differences between unity and the reciprocals of all the primes not exceeding a given number, stands in the relation of about 4 to 5 to the condensed portion consisting of the reciprocals of the natural numbers.

I will now proceed to establish similar inequalities relating to prime numbers of the respective forms $4n + 3$ and $6n + 5$.

Beginning with the case $4n + 3$, I shall use q_j to signify the jth in the natural succession of primes of the form $4n + 3$, and $q_{N.q}$ to signify the highest q not exceeding N, N.q itself signifying the number of q's not exceeding N.

[1] N p itself of course denotes in the above notation the number of primes (p) not exceeding N.