that the cosine of the angle between $a\,b\,c$, $a\,b\,d$, the faces of a tetrahedron, will be the determinant—

$$\begin{array}{cccc} \bullet & a\,b^2 & a\,d^2 & 1 \\ a\,b^2 & \bullet & b\,d^2 & 1 \\ c\,a^2 & c\,b^2 & c\,d^2 & 1 \\ 1 & 1 & 1 & \end{array}$$

divided by sixteen times the product of the faces $a\,b\,c$, $a\,b\,d$.

Or, again, if $a\,b$, $c\,d$ be any two non-intersecting edges of the tetrahedron, $\pm\, 2a\,b\,.\,c\,d$ cos $(a\,b, c\,d)$ ought to be equal to—

$$\begin{vmatrix} a\,c^2 & a\,d^2 & 1 \\ b\,c^2 & b\,d^2 & 1 \\ 1 & 1 & \bullet \end{vmatrix}$$

and as a matter of fact the cosine between $(a\,b, c\,d)$ is equal to $\dfrac{a\,d^2 + b\,c^2 - a\,c^2 - b\,d^2}{2a\,b}$.*

Again, if $a\,b\,c$, $d\,e\,f$ are any two triangles in space of 5, 4, or 3 dimensions the product of their areas into the cosine of their inclination will be a numerical multiple of the bordered determinant of the group $a\,b\,c$ in regard to $d\,e\,f$, and if they lie in the same plane their product itself will be that numerical multiple.

Similarly for two groups of four points lying in one space (as *ex gr.* that in which *we live, move, and have our being* †) the product of their bordered self-determinants will be equal to the bordered determinant of either group in respect of the other, because their *niveaus* coincide, and if we take two groups of five points each in ordinary space it again follows from the theorem that the bordered determinant between them must vanish, a statement which when the two groups coincide reverts to Cayley's condition concerning the mutual squared distances of five points in ordinary space.

Finally, there can be little doubt, I think, of the truth of the following theorem dealing with determinants (but unbordered) ‡ of which the general theorem we have been considering which deals with bordered determinants must needs be a corollary.

By $P : Q$ where $P$, $Q$ are two groups of $n$ points each, let us understand the determinant formed by taking the cosines of the angles which the $n^2$ lines connecting $P$ and $Q$ subtend at a point $O$ equidistant, in space of the necessary number of dimensions, from each of the $2n$ given points, and let $P'$, $Q'$ mean the groups $P$ and $Q$ augmented by the addition of $O$ to each of them, the theorem is that—

$$\cos\,(P', Q') = \frac{P : Q}{\sqrt{(P : P)(Q : Q)}}.\S$$

* Obviously therefore we can express the squared shortest distance between two non-intersecting edges of a tetrahedron as a rational function of the squares of all six. The formula in the text is well known and easily proved for the case of $a\,b\,c\,d$ being in a plane, which is enough to show that it must be true universally, for if we make $B\,C\,D$ rotate about $B\,C$, the projection of $C$ upon $B\,D$ does not move, and consequently $A\,C$ into the cosine of $A\,C$, $B\,D$ is invariable.

† It would perhaps be more correct to say "*which has its being in us.*"

‡ From which it follows that every algebraical theorem regarding square matrices expressed in the umbral notation is immediately convertible into a proposition in universal geometry; the umbræ cease to be mere abstractions, and acquire a *local habitation and a name* as points in extension.

§ $\sqrt{P : P}$ is in fact the factorial of $n$ divided by the $n$th power of the distance of $O$ from each point in $P$ into the content of (what I call) the *plasm* (of order $n$) denoted by $P'$.

A *plasm* of the order 1, 2, 3 means a rectilinear segment, a triangle, a tetrahedron—whence it is easy to deduce and define in exact terms the meaning of a plasm of any order as a figure bounded by plasms of the order next below its own. The squared content of a triangle is equal to the sum of the 3 squared contents of its projections on mutually perpendicular planes in ordinary space: but also to the sum of the 6 squared contents of its projections on 6 such planes in extension of 4 dimensions and so on—and in general the square of the content of a *plasm* denoted by $n$ points is similarly resoluble into a sum of $\dfrac{n(n-1)\ldots(n-i+1)}{1\,.\,2\ldots i}$, such squares in extension of $(n+i-1)$ dimensions; as these squared contents are all expressible immediately by Cayley's theorem in terms of squared distances, the above statement gives rise to a far from self-evident theorem in determinants. What I have here termed *plasms* might with more exactitude be termed *protoplasms*, as being the elements into which all other figures are capable of being resolved.

Thus for the case of $n$ equal to 2 if $O$ is the centre of the sphere passing through $a, b, c, d$, we ought to find the cosine of the angle between the arcs $a\,b$, $c\,d$ equal to

$$\begin{vmatrix} \cos a\,c & \cos c\,d \\ \cos b\,c & \cos b\,d \end{vmatrix}$$

divided by a square root of

$$\begin{vmatrix} \cos a\,a & \cos a\,b \\ \cos b\,a & \cos b\,b \end{vmatrix}$$

into a square root of

$$\begin{vmatrix} \cos c\,c & \cos c\,d \\ \cos d\,c & \cos d\,d \end{vmatrix}$$

*i.e.* equal to

$$\pm\,\frac{\cos a\,c\,.\,\cos b\,d - \cos a\,d\,\cos b\,c}{\sin a\,b\,.\,\sin c\,d}$$

as is the case.

There ought also to exist analogous theorems applicable to non-equi-numerous point groups depending in some way upon the minors of a corresponding rectangular matrix.*

J. J. SYLVESTER

New College, Oxford, April 1885

* It may be objected that the theorems of the text applied in their full generality beyond the limits of empirical space cease to affirm a relation between two different things and therefore lose their efficacy as such and become mere *definitions* of the meaning of the inclination of two figures in supersensible space. To meet this objection it is sufficient to give a general method for determining algebraically the projection of a point in space of $n$ dimensions on the niveau of $\nu$ points where $\nu$ is any number not greater than $n$; this it is easy to see may be effected as follows:—

($\alpha$) I observe that the niveau of any $\mu$ given points in a space of $n$ dimensions may be expressed in Cartesian co-ordinates by means of equating to zero each of $n - \mu + 1$ independent minors of a rectangular matrix containing $n + 1$ columns and $\mu + 1$ lines, the formation of which is too obvious to need stating in detail.

($\beta$) In order to project orthogonally a point whose $n$ co-ordinates in a space of $n$ dimensions are $x', y', \ldots z'$ upon a niveau (of the $(n-1)$th order) passing through $n$ given points defined by the equation $A\,x + B\,y + \ldots C\,z + L = 0$, we have only to write $x - x' : y - y' : \ldots : z - z' :: A : B : \ldots : C$, and combining the $(n-1)$ equations contained in this proportion with the given equation, the resulting values of $x, y, \ldots z$ determine the projection of the given point on the given niveau.

If now $\nu$ points are given in a space of $n$ dimensions and the projection is required of a given point upon their niveau we may proceed as follows:—

(1) Find the $n - \nu + 1$ equations which define the niveau.

(2) On each of the niveaus of the $(n-1)$th order which correspond thereto respectively find the orthogonal projections of the given point.

(3) Through these $n - \nu + 1$ projections of the given points and the given point itself draw a niveau which will be defined by $(n+1) - (n - \nu + 2)$, *i.e.* $\nu - 1$ equations.

Finally, combining these with the $n - \nu + 1$ original equations we have $n$ equations in all, and these will serve to determine the $n$ co-ordinates of the projection required.

This method is not always the most compendious, but is always sufficient, and enables us to attach a definite meaning to the inclination of two spaces of any the same order to one another: thus *ex. gr.*, the content of the projection of $a\,b\,c\,d$ on $e\,f\,g\,h$ divided by the content of $a\,b\,c\,d$ itself is the cosine of the inclination of the niveaus $a\,b\,c\,d$, $e\,f\,g\,h$, and the projections of the several points $a, b, c, d$ on $e\,f\,g\,h$ (say $a', b', c', d'$) being found by the preceding method, the content of the tetrahedron $a'\,b'\,c'\,d'$ (and therefore the inclination of the two niveaus) is a known quantity.

## GRESHAM COLLEGE

THE question of what is to be done with one of the greatest of existing London abuses, Gresham College, has again come up in connection with a letter from a "Londoner" in the *Times*. The *Times*, in a somewhat incomplete leader, animadverts strongly on the abuse, and urges its prompt remedying. Surely when the fact that London has no university in the true sense is attracting so much attention and the movement to supply the want is so powerful, it is absurd to allow the funds to be worse than wasted which represent the wreck of those which were originally intended for the maintenance of a real institution of this class. There were once 20,000 students at Gresham College, and when London does have a university, as it must have some time, even Gresham College will be without *raison d'être*.

"Topographically," the *Times* says, "the lecture-rooms are off the track of students. None of the apparatus of systematic instruction, in the way of examinations, accompanies the courses. Provision does not exist, or, at any rate is not employed, for the contact of the mind of the learner with the mind of the teacher. The lecturer ascends to his chair, recites or reads his stipulated discourse, and disappears with the mechanical routine of an automaton. The professorial staff, it might have been added, has as little internal unity as relationship to its classes. It is a concourse of atoms with no affinity except equality of stipends. To call the foundation a college is to use a manifest misnomer. It is as much a college as one at Oxford or Cambridge would be with the undergraduates and fellows suppressed, and the Master, Dean, Bursar, and Butler left to perpetuate the tradition. The Corporation of the City and the Mercers' Company are Sir Thomas Gresham's trustees, and derive very substantial advantages from his bounty.

"The inutility of the Gresham Lectures was recognised in the days of Dr. Johnson. Johnson lamented as bitterly as our correspondent that the able professors of Gresham College, which was 'intended as a place of instruction for London, contrived to have no scholars.' His explanation was that the professors lectured gratis, and grew indolent from the absence of pecuniary incentives to intellectual exertion. 'We would all,' he exclaimed with conviction, 'be idle if we could.' Permission to charge sixpence a pupil for each lecture would, in his opinion, have infused vitality into the institution; every professor would forthwith have grown 'emulous to have many scholars.' There could be no harm in administering his specific now. The good of a condition such as Gresham College has been reduced to is that any experiments may be tried upon it without excessive risk. But the failure of the foundation arises from deeper sources than those to which Johnson attributed it. Several of the present lecturers are notoriously of a temper and standing not to need a money bribe to urge them to do their duty. The Dean who is the Divinity Professor delights in occasions for ecclesiastical exegesis. He would rejoice to find a way of gathering five hundred receptive hearers to listen to the theological expositions he throws away on a meagre fraction of the number. Another Dean was Senior Wrangler, and is abundantly competent for the geometrical themes he has to discuss. The subject of civil law is committed to a most capable jurist. The Professor of Music is able elsewhere without any endowment to attract to his classes a large paying audience. The blame, as our correspondent concedes, does not lie with the lecturers, who only slumber in concert with their classes and their patrons. It must be imputed to the gross contempt which has been shown for all the conditions of educational success. Their founder intended his seven professors to be professors in a College which he did not survive to create. He died at the age of sixty, still immersed in public affairs, and before attaining the leisure for carrying out his idea of an 'epitome of a University in London.' Accidents for which it would be useless to condemn his trustees would have prevented them, had they otherwise been well disposed, from accomplishing his ambitious programme. His estate, so far as it was appropriated to the purpose, proved insufficient for the complete endowment of a College and its staff. A collection of lectures was left as it were in the air. For a time they appeared to have procured favour in spite of their disadvantages. In the nature of things they could not keep it permanently. They were without soil to take root and sprout in. The error of all concerned has been that the want was not supplied by incorporating either them in something else or something else in them. Last century was a period of educational, though not of intellectual, stagnation. Gresham College only languished in company with many other Colleges better furnished with the gifts of fortune. The present age has witnessed a revival of zeal for instruction by methods in which the Gresham foundation might have been turned to the greatest service, and has been turned to none. While London, and, most of all, the City, was careless of learning, it was no reproach to the managers of Sir Thomas Gresham's bounty that they converted it to no account. The absurdity is that for years the town, from its centre to its outskirts, has been crying out for educational appliances, and that Gresham College is suffered to remain as futile and superfluous as ever. Half-a-dozen institutions have been erected in or by the City to effect the objects for which Sir Thomas designed his foundation. For any one of them it would have been the most admirable nucleus; it would have afforded a starting point, and have bestowed the dignity of old descent. Thus it would have gained at last the reason for existence it has been craving in vain for a couple of centuries.

"Tastes of benefactors in distant ages do not always agree with the popular inclinations of the present. Reluctance on the part of trustees to deviate from the will of the men they represent is to be excused, though it cannot always be allowed to block the road to reform. When, however, a founder has let posterity into his confidence, and the application of his gifts clearly conflicts with his own views, it argues strange perversity or default of mental elasticity not to perceive where genuine respect for his wishes should lead. Without a framework in which they could be set and mutually co-ordinated, the Gresham Lectures cannot possibly do what the founder desired them to do. The public spirit of the City would not refuse to take up and finish the work which Gresham sketched out if it could be secure that his original instalment of beneficence was no longer wasted as now. Already it has been endeavouring to fill up the gap by its own exclusive exertions. The City of London College, the courses of the University Extension Society, lectures at the London Institution, the Technical College, Middle Class Schools, and not a few institutions besides, are spontaneous efforts of the past dozen years to work out the original idea of Sir Thomas Gresham. The proper City of London College is Gresham College. Around it as the centre all the other educational instruments of the City ought naturally to group themselves. Not the most punctilious conservatism could reprobate the Corporation and the Mercers' Company if they would use the authority they possess, and seek fresh authority, to aid in the promotion of that general result. Gresham College, as it is, has been for centuries, and is doomed to be, a burlesque of collegiate life. Its lectures must be equally dead whether delivered in a dead or a living tongue. Its choice is between becoming something more or something less than it is now. If it cannot develop, it had better cease to be."

## *ELECTRICITY AT THE INVENTIONS EXHIBITION*

THE International Inventions Exhibition is intended to illustrate the progress of invention during the period that has elapsed since the last Great International Exhibition in this country in the year 1862. Accordingly we find under Group XIII. electricity ranged under twelve classes, entitled respectively, generators, conductors, testing and measuring apparatus, telegraphic and telephonic apparatus, electric lighting apparatus, electro-metallurgy and electro-chemistry, distribution and utilisation of power, electric signalling, lightning-conductors, electro medical apparatus, electrolytic methods for extracting and purifying metals, electrothermic apparatus. Under such a classification there is no doubt that the Exhibition might have been made thoroughly representative of the wonderful progress that has taken place in this branch of science, both in its theory and practice, during the last twenty-three years. The reason that it is not so is twofold: electricity has had of late years many exhibitions dedicated to itself—those of Paris, Vienna, and Sydenham; and it was quite impossible in such an exhibition as the Inventions, where so