however, only a few authors have attached any importance. On investigation Mr. Cheyne found that the tubercle-bacilli were, unless when present in large numbers, only found in or among these epithelioid cells, and that the tuberculous nodules first begin by the entrance of bacilli into these cells and the subsequent development of the epithelioid elements. Surrounding these epithelioid cells a slight amount of inflammation occurs, giving rise to the small-celled growth around the tubercle, which is generally regarded as the growing part of the tubercle. This Mr. Cheyne denies, asserting that it is merely inflammatory tissue, and that the essential elements of the tubercle are the epithelioid elements in its centre. In the lungs these cells seem to be derived from the alveolar epithelium, in the liver often apparently from the liver cells, but in other organs and also sometimes in these from the endothelium of the lymphatics and blood-vessels.

In phthisis the bacilli were found at the margin of cavities and in the epithelioid cells surrounding the cheesy matter. Mr. Cheyne concludes that in phthisis the bacilli, inhaled into the alveoli, develop in the alveolar epithelium, cause accumulation of epithelial cells in the alveolus, and inflammatory hypertrophy of its walls. Thus the bacilli are practically shut off from the circulation and acute general tuberculosis cannot occur. The two extremes of phthisis are considered—the very rapid form or caseous pneumonia, and the slow form or fibroid phthisis. In the former the bacilli grow rapidly, are fairly numerous, and the lung rapidly breaks down; in the latter the bacilli grow slowly and with difficulty, and hence extensive fibrous formation occurs.

There are many other points of interest in this research to which we cannot allude, but which will be found at length in the Report. The Association is to be congratulated on having chosen such a fertile subject for their first report, and we hope that they will continue to encourage similar work.

PROFESSOR H. J. S. SMITH AND THE REPRESENTATION OF A NUMBER AS A SUM OF SQUARES

THE award of the great Mathematical Prize of the French Academy to the late Prof. H. J. S. Smith may have the effect of drawing the attention of mathematicians to the wonderful extent and value of his researches on the Theory of Numbers. Probably no more important or remarkable mathematical investigations have ever appeared in this country than his memoirs on systems of linear indeterminate equations and congruences and on the orders and genera of ternary quadratic forms and of quadratic forms containing more than three indeterminates, which were published in the *Philosophical Transactions* for 1861 and 1867 and the *Proceedings of the Royal Society* for 1864 and 1867. The results contained in these papers are by far the greatest additions that have been made to the Theory of Numbers since it was placed on its present foundation by Gauss in the "Disquisitiones Arithmeticæ." The subject for which the prize was awarded to Prof. Smith was that of the theory of the representation of a number as a sum of five squares, and of this question as well as that of the corresponding one for seven squares he had given the complete solution in the *Proceedings of the Royal Society* for 1867 (vol. xvi. p. 207). The words with which Prof. Smith introduced his statement of the solution of these important questions are as follows:—

"The theorems which have been given by Jacobi, Eisenstein, and recently in great profusion by M. Liouville, relating to the representation of numbers by four squares and other simple quadratic forms, appear to be deducible by a uniform method from the principles indicated in this paper. So also are the theorems relating to the representation of numbers by six and eight squares, which are implicitly contained in the developments given by Jacobi in the 'Fundamenta Nova.' As the series of theorems relating to the representation of numbers by sums of squares ceases, for the reason assigned by Eisenstein, when the number of squares surpasses eight, it is of some importance to complete it. The only cases which have not been fully considered are those of five and seven squares. The principal theorems relating to the case of five squares have indeed been given by Eisenstein (*Crelle's Journal*, vol. xxxv. p. 368); but he has considered only those numbers which are not divisible by any square. We shall here complete his enunciation of those theorems, and shall add the corresponding theorems for the case of seven squares."

In the announcement of the subject for the prize in the *Comptes Rendus* in February of last year, reference was made to the work of Eisenstein, but the fact that his solution had fifteen years before been completed by Prof. Smith—who had also solved the problem in the case of seven squares, the whole being only a corollary from the general principles contained in his memoirs—seems to have escaped the attention of the proposers of the subject. In the paper in the *Proceedings of the Royal Society* the results only for the case of five squares and seven squares are given, the demonstrations being omitted; and accordingly, when the subject for the prize was announced, Prof. Smith followed the only course open to him, and communicated to the Academy his demonstrations for the case of five squares.

All who knew Prof. Smith will understand how uncongenial to him was the idea of becoming a competitor for the prize, but under the circumstances he had no choice. It is a singular tribute to Prof. Smith's mathematical powers, as well as a curious episode in the history of mathematics, that the French Academy should have chosen as the subject of the "Grand Prix"—thereby indicating their opinion of its importance in the advance of the science[1]—a question that had been solved already fifteen years before as a corollary from more general principles.

The state of the question of the number of ways in which a number can be expressed as a sum of squares therefore stands as follows:—For two squares the solution was given by Gauss in the "Disquisitiones"; the cases of four, six, and eight squares are due to Jacobi, Eisenstein, and Prof. Smith (see *Report of the British Association for* 1865, p. 366). In these cases in which the number of squares is even, the problem can be solved by means of elliptic functions, and it is not necessary to have recourse to the special methods of the Theory of Numbers; but it is not so in the case when the number of squares is uneven, and the question is then essentially "arithmetical" as regards its method of treatment and expression. The case of three squares was given by Lejeune-Dirichlet, and is included in Prof. Smith's general treatment of ternary quadratic forms in the *Philosophical Transactions* for 1867: the enunciations for the cases of five squares and seven squares were given, as has been stated, in the *Proceedings of the Royal Society for* 1867. The demonstrations for the case of five squares have been communicated to the French Academy, but those for seven squares still remain unpublished in Prof. Smith's note-book. This class of questions ceases to admit of the same kind of solution when the number of squares exceeds eight, so that with the publication of the demonstrations for seven squares the solution of the whole problem will be complete. It will be seen that Prof. Smith has had a large share in this great mathematical victory.

[1] "L'Académie était donc fondée à espérer que ce voyage de découvertes imposé aux concurrents à travers une des régions les plus intéressantes et les moins explorées de l'arithmétique produirait des résultats féconds pour la science. Cette attente n'a pas été trompée." Report on the award of the prize, *Comptes Rendus*, April 2, 1883. In this report however no mention is made of the fact that these "résultats féconds" had been published in 1867.

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