14 lbs.) and by the nasal bone not diminishing so rapidly in size. The mountain fox (*Vulpes lagopus*, L.) is very common. The common fox (*Vulpes vulgaris*, Gray) appears also to be common. A red fox, shot in October, differs considerably from the common, and approaches the mountain fox in several particulars. The fox's food during winter appears to consist of hares, ptarmigans, and lemmings. Of lemmings three species were met with, *Myodes obensis* (the most numerous), *M. torquatus*, and *Arvicola obscurus*. The Tchuktches state that a little mouse also occurs, which Nordquist supposes to be a *Sorex*. The two lemmings often showed themselves above the snow during winter, which was not the case with *Arvicola obscurus*. The wolf was seen only twice. The wild reindeer was also uncommon, traces of them having been seen only once. Traces of the land-bear were also seen, and the natives stated that they were not uncommon in summer. The marmot (*Arctomys*) occurs in abundance. An animal described by the natives as living by the banks of streams is supposed to be the common otter. Two weazel-skins were obtained from the natives. It is not certain whether the ermine occurs there. Only two marine mammals have been seen during the winter, the Polar bear and the ringed seal (*Phoca fœtida*). The latter is caught in great numbers, and along with fish and various vegetables forms the main food of the natives. Of land birds there winter in the region only three species, viz., *Strix nyctea*, *Corvus corax*, and *Lagopus subalpina*. The last-mentioned is the most common. On December 14 two large flocks of ptarmigan, one numbering about fifty, were seen about ten miles from the coast. The raven is common at the Tchuktch villages. Its first egg was obtained on May 31. The mountain owl was seen for the first time on March 11, but according to the natives, it is to be met with all winter. In open places on the sea there occur during winter, according to the natives, two swimmers, *Uria Brünnichi* and *Uria grylle*. Besides these there possibly winter on the sea a species of *Mergulus* and one of *Fuligula*, a specimen of the former having been obtained on November 3, and of the latter on March 9.

(*To be continued.*)

GALILEO AND THE APPLICATION OF MATHEMATICS TO PHYSICS[1]

TWO hundred and ninety-eight years ago to-day (November 5, 1581) Galileo Galilei, then a boy between seventeen and eighteen, matriculated as a medical student in the University of Pisa. At that time Medicine was perhaps the least satisfactory of scientific studies, and though his family had influential professional connections, the empirical maxims and the semi-metaphysical reasons by which they were supported never caught the young man's fancy or satisfied his intellect. We first hear of him listening outside the door in which Ricci, the Court mathematician of Florence, who happened to be spending some time at Pisa with the Grand Duke, taught the pages a little Euclid. For a couple of months Galileo neglected his medicine, and greedily absorbed his Euclid through the key-hole till he found some chance opportunity of introducing himself to the Professor, who was delighted with his new pupil. Ricci presented him with a volume of Archimedes, and the great mathematician and physicist of Syracuse became the spiritual father of the young Italian student. In spite of the straitened circumstances of his family, and the chances of fortune that awaited him in a decorous prosecution of his regular medical studies, he deserted them, and attached himself to Ricci.

Watching one day the long swing of a lamp hung from the roof of a church, we are told that he noted the times it took in oscillation after oscillation, and found that though the arc through which it swept died down till it was scarcely visible, the time it took from each farthest right hand point to the succeeding farthest left hand point of its sweep was always the same. He applied the knowledge he had gained at once to the more accurate measurement of the regularity of the pulse beats. The observation of the student, and the immediate practical application of it, was the sure forerunner of the greatness of the man. He knew that Science is Measurement three centuries before Comte laid it down as the definition of mathematics, or Marks had been born to caricature the maxim in his diploma picture.

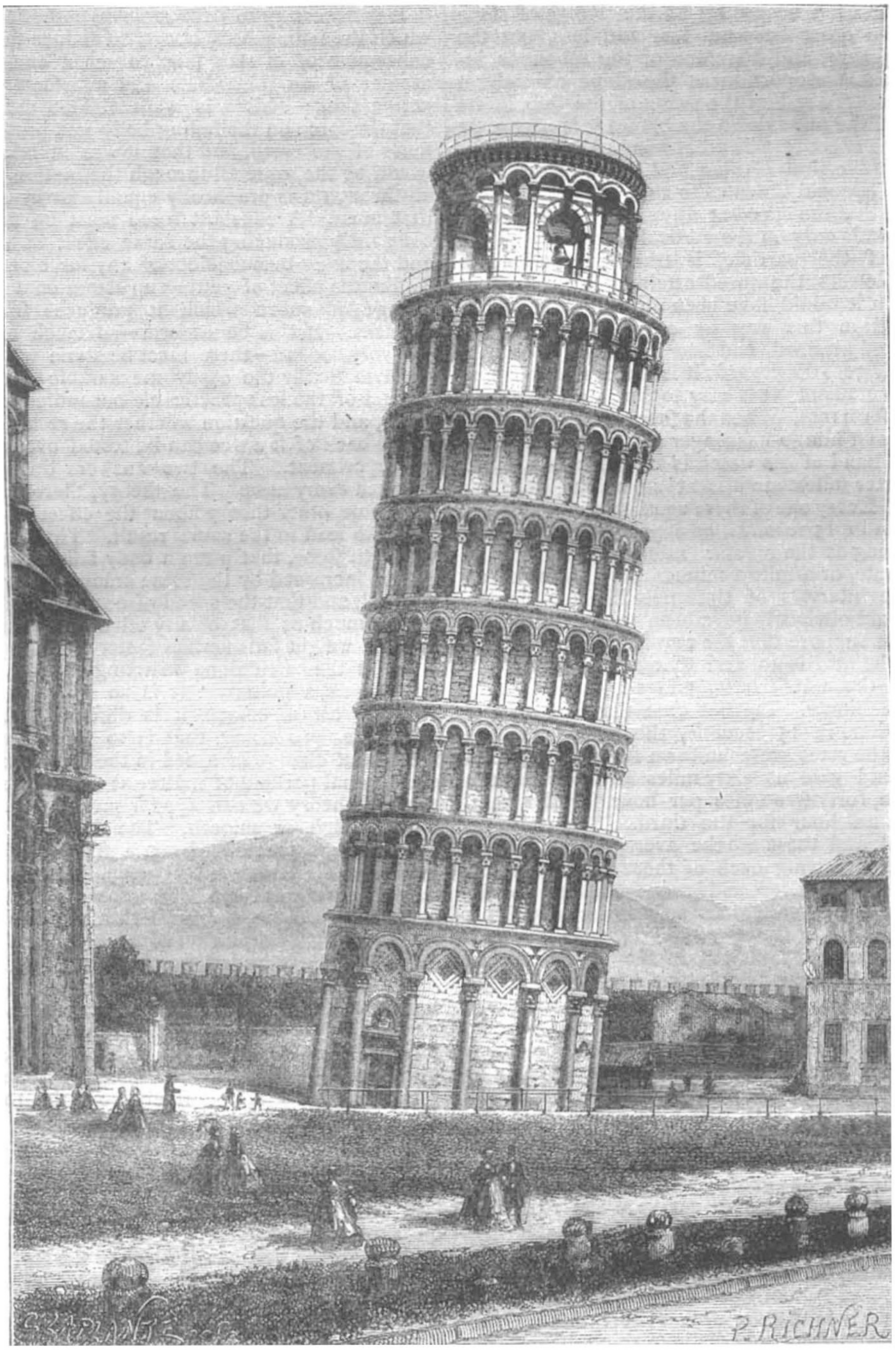
At that time the Peripatetic philosophy was dominant over Europe, and tyrannized in Italy. The followers of Aristotle naturally travestied the errors of their master. In his own time Aristotle was a genuine observer of nature, and, as Galileo afterwards said of him, he would have been the last to dispute a fact because it contradicted his preconceived opinions. His followers, who were not observers, had constituted a universe on high *a priori* principles. They taught that there were two great classes of things perishable and terrestrial, one heavy, tending by an irresistible law of their essential nature to the mathematical centre of the universe, the other light, and tending irresistibly away from it. Things imperishable and extra-terrestrial moved by a like necessity in everlasting circles round the centre of all things. A body of 2 lbs. weight, having more tendency to the centre than a body of 1 lb., must fall faster, and acquire a greater velocity in an equal time. With *a priori* principles like these observation was superfluous. Galileo questioned them and put them to the *examen rigorosum* of experiment. The explanation of the isochronism of the larger or smaller swings of the pendulum lay in the fact that though when the moving lamp started from a higher point it had further to fall—it began to fall more nearly perpendicularly and faster, and it swept through its larger arc with a greater velocity at every point. When he took two such pendulums of equal length, to the end of one of which a lamp weighing 1 lb. was fastened, and to the end of the other a weight of 2 lbs., Galileo found that their times of oscillation were the same.

The Peripatetic dictum of the greater gravity of heavier bodies was in contradiction with this simple fact. Galileo took the two weights to the top of the hanging tower of Pisa, and let them fall. They fell at the same or practically the same instant. Though the simultaneous thud of these two weights on the ground was the death-blow of the Peripatetic assumption, it was not enough to convince teachers who had grown grey in teaching it. But a moment's thought now will serve to show us not merely that it is so, but why it must be so. Instead of the mass of 2 lbs., imagine for a moment that the 2 lbs are made up of two single pound weights, each identical in shape and material with the other mass of 1 lb., and that all three drop together. All three will come to the ground together. If the two pound weights are made to adhere to each other by ever so thin a film of glycerine, there will be no strain on the film, and they will not separate If an imaginary section is cut through a single mass of two pounds there will equally be no strain or shearing force along that section. The tendency of the two single lbs. downwards is twice as great as that of the 1 lb., but it has to move two masses instead of one. Ten runners who keep abreast of each other do ten times the work of an eleventh runner on the other side of the course. Man for man, each does the same work, and each man's work has the same effect in producing the racing speed of each. An imaginary or real thread might tie the ten together, but there would be no strain on the thread, which would not snap, if their rates of running were the same.

Galileo often returned to the pendulum, and completely

[1] An Introductory Lecture, by William Jack, M.A., LL.D., F.R.S.E., Professor of Mathematics in the University of Glasgow, formerly Fellow of St. Peter's College, Cambridge.

established the laws of its motion in ordinary small oscillations. He showed that though the weights at the end of the string have no effect on the times of oscillation the length of the string has, that these times are twice as long for a string four times as long, three times for one nine times as long, and always in the proportion of the square roots of the lengths. In proving this he had to investigate motion along a slope or Inclined Plane, and it was he who first showed that whatever the incline, the speed acquired by a body moving on it depends not on the amount of



The Leaning Tower of Pisa.

ground it has covered on the plane itself, but on the vertical drop between its starting point from rest, and its position at any moment. The pendulum moves along an arc of a circle, and something very like that arc would be got by drawing instead of it small chords of the circle from point to point in it successively. The smaller these successive chords become the nearer the sum total of them is to the arc, and the motion of a heavy particle constrained to move down them is substantially the same as that of the bob at the end of the pendulum. These successive chords are so many inclined planes, and the movement of the weight down the entire series, is identical with the swing of the bob in the arc. More is necessary to establish this completely than Galileo was able to supply. In passing from plane to plane the particle must be supposed to make a slight rebound at each, a rebound which is less for each, according as the change of slope from one to the other becomes less and less, but the number of the planes, and therefore of the rebounds, increases in the same proportion as the slope of each to each diminishes. To reduce the swing of the bob in its arc to the fall of the mass down the planes it is necessary to show that the effect of this great number of small rebounds is negligible, and Galileo had not advanced far enough in the Fluxional Calculus to show it.

The principle that the speed at any point of the downward slope depends only on the vertical drop between the two positions of the particle, is true independent of friction which lowers the speed attained in a constant proportion. But it would have been difficult to establish the truth stated in this way by ordinary experiment. What is the speed attained, and how are we to recognise it? As the body goes downwards it is increasing in speed from moment to moment. It is easy to time a railway train running at a uniform rate. When the first quarter milestone he notices flies past him, a passenger sees, let us suppose, that the second hand of his watch is at 5 seconds, while at the next quarter milestone it is at 20, at the third 35, at the fourth 50. Every one of these equal intervals is swept over by the train in 15 seconds, or a quarter of a minute. The train is going at the regular rate of a quarter mile per quarter minute, or a mile a minute, or sixty miles an hour. Had the intervals of time noted been different, the problem would obviously have been much more complicated. Let us suppose that the two first 5 and 20, are as before, that the next is 40, and that at the fourth the second hand of the watch has again come round to 5 seconds past the minute. In that case the first quarter mile interval is done in 15 seconds, the next in 20, the third in 25. If the rates were uniform for each interval these figures would give us sixty miles an hour for the first quarter mile, forty-five miles per hour for the next, thirty-six miles an hour for the third. The train is slackening speed, and these are the average rates during the time spent in covering each of these quarter miles. But the train does not drop suddenly from one to the other, and nothing in nature does so. Point by point it has a different rate, and the question, What is the rate at any point? is not easily answered. How, then, are we to measure the rate of speed at a point when that rate is constantly changing? We must seek some necessary consequence of any law of change which we suppose, and we must transform the question, the answer of which it is difficult to verify, into one which it will be easy to subject to an experimental test Galileo appealed to mathematics, and showed that if his theory, that the velocity depends on the vertical drop, be true, the amount of the vertical drop must be four times as great for two seconds, and nine times as great for three seconds, as for one second, and he set himself to compare the real with the theoretic result.

Let us consider what seems a simple thing, a fall in space, where there is no inclined plane at all. What is the amount of fall for so many seconds? The difficulty in answering accurately is that for even a short time the fall is very large. It is of no use distinguishing between a fall of 16 feet, for instance, and one of 20¼ feet, if the times of description, which are 1 second and 1⅛ second, are too nearly the same to be distinguished by our measurement of time. In Galileo's day the measurements of time were only beginning to be a little delicate, chiefly through his own discoveries, and an error of ⅛ of a second in measurement is obviously easy to make, when one of 4 feet is not easy. In the simpler case of free fall, therefore, Galileo could not compare spaces and times conveniently, because his measures of space were so much more accurate than those of time. The experimental test can be more readily applied to the inclined plane because the fall is slower and there is no other vital alteration in the conditions of the problem.

It is necessary to form some hypothesis about the law which the falling body obeys, to deduce the mathematical consequences of that law, to select one of them which admits of an immediate and satisfactory experimental verification. This was what Galileo did. He believed that the force on the falling body was probably due to the mass of the earth, and that it was at least likely that it would be the same all through the motion, as the particle all through it is practically equally far from the centre of that mass. A constant force must be measured by its constantly producing the same effect in the same time, and the first obvious effect of any force on a falling body is, like the effect of getting up steam on a locomotive, the change of speed which it produces from moment to moment. If this be uniform—so much extra speed put on every second—there must be some way of connecting mathematically the easily measurable spaces and times instead of the less practicable but more direct speeds and times, and the question whether the result and the theory at the back of it agree can be tested over and over again by experiment. The two answers do agree, and they agree in every case. The theory, therefore, is right, unless some other theory about the effect of forces can be found to lead to the same result. The hypothesis about the earth force, that when a body falls from rest its speed will be increased by the same amount in every equal time interval, and that the speed of any body will be increased just as much as that of any other, is a true hypothesis. A 10 lb. weight falls neither faster nor slower than a 1 lb. one. If the earth alone be acting on both, a feather falls as fast as a guinea. It is so in vacuum, though in ordinary air, of course, it is different. A force always the same, producing, that is to say, always the same amount of change of speed in the same time, is acting on every equal particle of matter at the earth's surface. To test this theory we can appeal practically to the inclined plane, rough or smooth. The force on a body falling along it at any moment bears a fixed proportion to that in a free fall; a very small proportion, if the plane has only a very slight slope. Obviously the length of the line along such a plane, down which a body runs in a second, is a very small proportion of that of the free fall in the same time. In the latter case, what to Galileo's power of measuring time was an almost imperceptible difference involved a very marked difference in the spaces gone through, so that it was difficult to verify the law. In the former the spaces needed to be measured for experiments lasting even a few seconds become reasonable. In three seconds a body falling freely from the top of a steeple 144 feet high would fall to the bottom, and it would only take five seconds to fall down Tennant's stalk, but it is easy to make a plane such that a body will only fall down 14 feet along it in three seconds.

It was in connection with his investigations of motion on a plane that Galileo laid down the principle that perhaps serves best as the basis of the theory of balancing forces, the principle of what is called Virtual Velocities. Every one is familiar with it, in the ordinary maxim, that what is gained in speed is lost in power. In the board laid across a fallen tree, on which children see-saw, the lighter child is put at the extremity of the longer arm. With a plank, 12 feet long, a child 50 lbs. weight will be balanced against one 70 lbs. weight when the plank rests on the tree 7 feet from the light child's end, and 5 feet from the heavy one's. When they swing, the amount of swing is proportional to the distances from the fixed point. If the plank moves, so that the child at the 7 feet end rises through seven inches, the other goes down through five. In every case like this, where forces are in equilibrium on a system, we can imagine a motion given, every point moving according to the geometrical circumstances. Let us imagine such a motion. When two forces act on a system and keep it at rest, multiply the space through which the point of application of each force moves, referred to the line in which the force acts, by the measure of the force. When there is equilibrium the resulting quantities are equal and of opposite signs. The one child weighing 50 lbs. rises vertically through 7 inches, and we may call the product 350 inch-lbs. upwards. The 70 lbs. child moves in the same time 5 inches downwards, and the product, which is 350 inch-lbs. downwards, is equal and opposite to the other. If there is equilibrium it must always be so; if it is so there must be equilibrium. It was to Galileo that we owe this most fruitful of statical principles. It can easily be extended to the case when any number of forces act at any number of points on a body or a system, but it was not till a century later that John Bernouilli could state it in all its generality, or show how admirably it serves as a sufficient basis for the whole theory of equilibrium.

These laws of falling bodies and of virtual velocities marked the greatest advance in mechanical science since the world began. The nature of the earth's common action on all bodies at its surface had, in fact, been ascertained. The question that had been put directly to nature had been completely answered, and the answer was final.

The Peripatetics had a singular notion of what they called Inertia. According to them, a body had a natural tendency to move at a given speed straight towards the centre of the earth if it were heavy, and straight away from it if it were light. The continuance of that natural motion, in that direction, at that speed was ensured by inertia. Strike the body in that or in any other direction, and an immediate change takes place, but it is a change which disappears if the body is moving in a vacuum. In ordinary air it is kept up, because the air behind, from which the body is suddenly taken away when it is struck, instantly closes up, and strikes it like a spring which has been let go. At every new position it leaves air, and air springs after it to keep it going. As far as it was then possible, Galileo worked out the consequences of this theory and those of his own, which was that stated in Newton's first Law of Motion—that except where any external force operates, motion in any direction at a certain rate will continue indefinitely in that direction at the same rate. The result was that the old theory was proved to be wrong. As with the first law of motion, so with the second. It is substantially this, that when a force acts on a particle in motion, it produces the same effect in changing that motion as it would if, before it began to act, the body were at rest. Suppose a particle moving with a speed which may be described as 10 feet per second northward and 8 feet per second eastward. Let a force suddenly act on it, the effect of which is to change its rate of going to 17 feet per second northward and 13 feet per second eastward. The amount gained is an addition of speed of 7 feet per second northward and 3 feet per second eastward. Imagine the same force acting on a particle identical with the former, but initially at rest. It will make that particle begin to move from rest at the same rate of 7 feet per second northward and 3 feet per second eastward which it gained in the former motion. The effect in changing rate has been the same as if the body had been at rest, and the whole effect on the eastward direction has been the same as it would have been had there been nothing to affect it in a northerly direction.

It was through the combination of these two principles that Galileo was able to solve another and more difficult problem. Until they were verified by the success of millions of predictions founded on them, they were not so much principles as theories or hypotheses. A fulfilled prediction of any complicated phenomenon raises the hypothesis on which it has been explained to the dignity of a probable truth. Let a bullet be started in an oblique direction at a certain speed—we can predict, by applying these two principles, the way in which it will move and the course it will follow. Let us take one which is sent off at a rate of speed compounded of 32 feet per second vertical and 20 feet per second horizontal. At every point of its path, it will keep both these rates except so far as gravity changes them, and gravity will do by it as a moving body just what it would do by a body starting from rest. To the latter it would give a downward speed of 32 feet per second in a second. In a second it will give just enough downward speed, therefore, to annihilate the upward speed of the bullet. After a second, it will have ceased to have any upward speed, but it will go on with the horizontal speed of 20 feet per second. In its first second the bullet has moved away from its starting-point 20 feet in a horizontal direction and 16 feet upward, because a fall of 16 feet from rest is needed to generate that velocity of 32 feet per second downward, which is wanted to destroy the upward velocity of the amount with which it started. At the end of the first second it has reached its new position by a certain path. Till the bullet comes to the ground again another second will suffice, during which it will fall through 16 feet vertically, and acquire a speed of 32 feet per second downward as it started with 32 feet per second upward, and it will move horizontally 20 feet further from the starting-point. When the second second closes, the particle has again reached the ground by a path which is the left-handed facsimile of that by which it rose.

There are thus three measurable things, all consequences of our fundamental laws. Does the bullet rise 16 feet? does it strike the ground 40 feet away from where it started? does it take 2 seconds to do it in? Nature answers that all these things are so. If we take some means of making the bullet record or picture its path on a board or paper we shall have a still completer answer to the question. Galileo's mathematics were enough to show him that if these two laws were true the curve described must be a parabola—except so far as it is slightly modified by the resistance of the air—and the parabola calculated is the parabola described. Such a proof is all but conclusive. Every point in the path really found has thus been predicted as the mathematical consequence of these two laws, and when this prediction is repeated and confirmed in every experiment, doubt vanishes, the laws are securely established, and the secret of nature has been found.

(*To be continued.*)

JAMES CLERK MAXWELL, F.R.S.

JAMES CLERK MAXWELL, whose premature death on Wednesday last week, science has to deplore, was born in 1831, being the only son of John Clerk Maxwell, Esq., of Middlebie. His grandfather was Captain James Clerk of Penicuik, whose two sons were the Right Hon. Sir George Clerk, Bart., of Penicuik, and the above-mentioned, John Clerk Maxwell. Captain James Clerk was a younger brother of Sir John Clerk of Penicuik, and on the death of the latter Sir George Clerk succeeded to the estate of Penicuik, while John succeeded to the estate of Nether Corsock, part of the Middlebie estate, which had come into the family through marriage in a previous generation with Agnes Maxwell. Along with this estate John Clerk assumed the family name of Maxwell. When James Clerk Maxwell was eight years old, his mother died, and his father, who had been called to the Scotch Bar, but never practised as an advocate, lived a retired life, devoting himself to the care of his estates, and of his son.