and on the linear transformation of the integral $\int \frac{du}{\sqrt{U}}$.—Prof. Clifford has an excellent paper on the canonical form and dissection of a Riemann's surface. Prof. H. J. S. Smith contributes the conditions of perpendicularity in a parallelepipedal system, and a very interesting presidential address on the present state and prospects of some branches of pure mathematics. Mr. Spottiswoode writes on curves having four-point contact with a triply infinite pencil of curves, and Prof. Wolstenholme gives an easy method of finding the invariant equation expressing any poristic relation between two conics.

LETTERS TO THE EDITOR

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts. No notice is taken of anonymous communications.*

[*The Editor urgently requests correspondents to keep their letters as short as possible. The pressure on his space is so great that it is impossible otherwise to ensure the appearance even of communications containing interesting and novel facts.*]

Trajectories of Shot

I HOPE you will be able to afford me space for a few remarks on the following extract from a paper on the Trajectories of Shot, by Mr. W. D. Niven, which appeared in the *Proceedings* of the Royal Society for 1877.

Mr. Niven arranges his paper under three heads, calling them the first, second, and third methods. The third method is the one he favours, while he endeavours to dispose of the other two in the following terms:—

"§ 11. It will be observed that the two foregoing methods each open with the same equation (a). Now there is a serious difficulty in the use of that equation. Suppose, for example, we were to integrate over an arc of 1°, we should have to use the mean value of k between its values corresponding to the velocities at the beginning and end of the arc. But we do not know the latter of these velocities; it is the very thing we have to find. The first steps in our work must be to *guess* at it. The practised calculator can, from his experience, make a very good estimate. Having made his estimate, he determines k. He uses this value of k in equation (a), and if he gets the velocity he *guessed* at, he concludes that he *guessed* rightly, and that he has got the velocity at the end of the arc. If the equation (a) does not agree with him, he makes *another guess*, and *so on till he comes right*."

The case would be indeed hopeless, if all this was quite correct. But I have to inform Mr. Niven that, in all proper cases v_β may be found *accurately* from equation (a), and without any "guessing" whatever. Taking Mr. Niven's own solitary example, I will *calculate* the value of v_β at the end of an arc, not of 1°, but of 3°, and compare my result with his own. The initial velocity, v_α, is here 1,400 f.s., and the corresponding value of the coefficient k_α, given in my table, is 104·0. Substitute this value for k in equation (a), given below, and v_β will be found 1291·7 f.s., a *first* approximation. Now *calculate* the mean value of k between velocities 1,400 and 1,290 f.s. by the help of the table, and it will be found to be equal to 106·3. Substitute this new value of k in equation (a), and v_β will be found 1289·8 f.s., a *second* approximation. We must stop here, because if we attempted to carry the approximation further, we should obtain the same value of k, and therefore of v_β, as in the second approximation. Mr. Niven finds $v_\beta = 1290$ f.s.

Of course in ordinary cases, a calculator, in making his first approximation to v_β, would commence by taking a value of k corresponding to a velocity *somewhat below* the initial velocity. In this way a better *first* approximate value of v_β would be found. Thus, again referring to Mr. Niven's own example, I will take a step over an arc of 6°, from $\alpha = +3°$ to $\beta = -3°$. The initial velocity is 1,400 f.s. I now go so far as to "guess" that the mean value of k will correspond to a velocity considerably below 1,400 f.s., and take $k = 107·9$, corresponding to a velocity 1,300 f.s. This gives $v_\beta = 1208·1$, a *first* approximation. The mean value of k between 1,400 and 1,210 f.s. is now found to be 107·2, which gives $v_\beta = 1209·0$ f.s. Mr. Niven obtains 1207·4 by stepping over two arcs of 3°. If any further adjustment was required, proportional parts might be used, seeing that a correction $\delta k = -0·7$ gives $\delta v_\beta = +1·8$.

Mr. Niven then proceeds to question the *accuracy* of what he is pleased to call the "process of guessing," as follows:—

"It seems to me, however, that this method of going to work, leaving out of account the loss of time, is open to objection in *the point of accuracy*. For, first there is no method of determining on what principle the mean value of k is found—what manner of mean it is. Again, let us suppose for an instant that the velocity at the end of the arc *guessed* at, and the value of k, are in agreement; that is to say, let the equation

$$\left(\frac{1,000}{v_\beta}\right)^3 \sec^3\beta - \left(\frac{1,000}{v_\alpha}\right)^3 \sec^3\alpha = \frac{d^2}{w}\,\frac{k}{g}(P_\alpha - P_\beta) - \text{(a)}$$

hold for the values of v_β and k used by the calculator. It by no means follows that he has hit on the right value of v_β and k. For if he is dealing with a part of the tables in which $\frac{dk}{dv}$ happens to be nearly equal to $-3\frac{wg}{d^2}\,\frac{\sec^3\beta}{P_\alpha - P_\beta}\,\frac{(1,000)^3}{v^4}$, it is obvious that there are ever so many pairs of values of v_β and k which will stand the test of satisfying the above equation. Now an examination of Mr. Bashforth's tables for ogival-headed shot shows that the value of k diminishes as v increases from 1,200 feet upwards, so that $\frac{dk}{dv}$ is negative for a considerable range of values of v which are common in practice. It is not at all unlikely, therefore, that the value for $\frac{dk}{dv}$ just stated may often be very nearly true; in which the case the *process of guessing* becomes extremely dangerous."

I here observe that Mr. Niven is not entitled to assume that, because two quantities have the *same sign*, they will therefore be probably often nearly of *equal value*. Without discussing the value of his test of danger, I have to state that my tabular value of $\frac{\delta k_\beta}{\delta v_\beta}$, for velocities above 1,200 f.s., lies between 0 and −0·09. I have calculated the numerical values of Mr. Niven's expression for $\frac{dk}{dv}$, for shot fired from various guns, from the Martini-Henry rifle up to the 80-ton gun, and have always obtained a numerical result so far outside the limits of the tabular value, that, for the present, I conclude that Mr. Niven's condition (whatever may be its value) *is never nearly satisfied in any practical example*. But when a practical case is produced where "ever so many pairs of values of v_β and k," *differing sensibly*, "stand the test of satisfying the above equation" (a), it shall receive my best attention.

It is well known that the problem of calculating the trajectory of a shot, like so many other practical problems, does not admit of a direct and complete solution. So that all solutions, being approximations, are more or less erroneous. But I feel perfect confidence in the results given by my methods of calculation, because, the smaller the arcs taken at each step, and the nearer the *calculated* will approach to the *actual* trajectory. But methods of approximation require to be used with judgment. For instance with the heaviest shot in use, we may take steps of 5° for velocities above 1,100 f.s.; while for small arm bullets arcs of half a degree will be quite large enough. In any case of real difficulty the remedy will be to divide the trajectory into smaller arcs.

From what I have said it appears that my method of finding the trajectories of shot, *when properly applied*, is neither a "process of guessing" nor yet "dangerous."

Minting Vicarage, March 8 F. BASHFORTH

Australian Monotremata

I AM surprised to find that "P. L. S." (vol. xvi. p. 439), was not aware that the Echidna *Tachyglossus hystrix*, is found in N. Queensland. For the benefit of your readers I may mention that the Australian Museum possesses a fine specimen of *T. hystrix* from Cape York. Mr. Armit, of Georgetown, Mr. Robt. Johnstone, and others, have frequently found them in various parts of Queensland. One specimen from Cape York was obtained there by our taxidermist, J. A. Thorpe, in 1867.

The Platypus (*Ornithorhynchus anatinus*) is also found in Queensland as far north as the Burdekin at least, perhaps further.

Tachyglossus, strictly speaking, has no pouch, but the *areola*