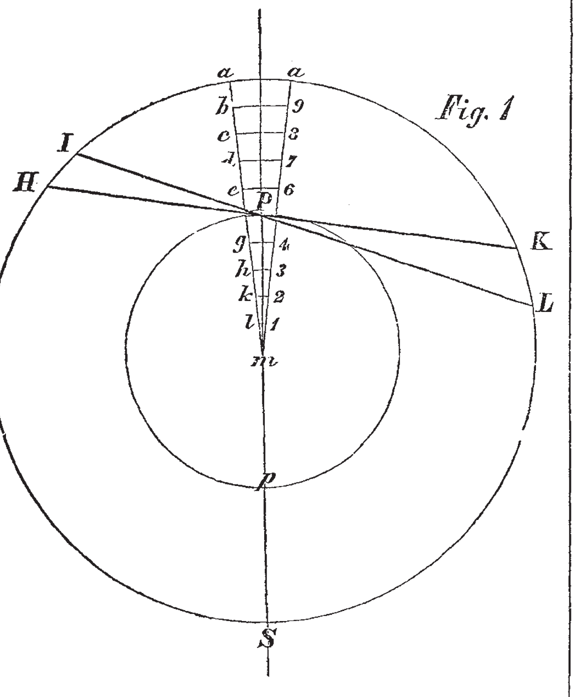
THE SOURCE OF SOLAR ENERGY

ALL incandescent bodies shrink rapidly if permitted to radiate freely, the rate being nearly proportional to the degree of incandescence. The enormous temperature maintained at the surface of the sun must therefore produce rapid shrinking, although we do not know the rate by actual observation. We know, however, what amount of mechanical energy the sun parts with in a given time, and we know the size and the specific gravity of the solar mass.

Demonstration is not needed to prove that motion of the particles within a spherical body towards the centre caused by attraction, develops a certain amount of mechanical energy resulting in the generation of heat within the mass. Nor is it necessary to show that the fixed relation between heat and energy enables us to determine the extent of contraction produced by gravitation, during cooling, if we can ascertain the amount of heat radiated in a given time by a sphere of known size and specific gravity. With reference to the sun, the elements thus specified are of the following magnitudes :—Heat radiated per minute, 312,000 thermal units from one square foot of surface; diameter, 852,584 miles; specific gravity, 0·250 compared



to that of the earth, or 5·50 × 0·250 = 1·37 of water. Hence assuming that the mass is homogeneous, the weight of one cubic foot of the matter composing the sun will be 62·5 × 1·37 = 85·6 pounds. It will be seen presently that, in case the sun's mass is not homogeneous, the want of homogeneity will not materially affect the question of attraction and the resulting energy. At first sight it would appear that no probable amount of contraction of the sun could develop by gravitation towards the centre an amount of dynamic energy of 312,000 × 772 = 240, 864,000 foot-pounds per *minute* for each square foot of the solar surface. Yet, so vast is the mass contained in a spherical pyramid, the base of which is one square foot and whose length is equal to the sun's radius, that a very small longitudinal contraction suffices to develop by gravitation towards the sun's centre the stated enormous dynamic energy. It will be readily understood that the energy developed by the shrinking of a spherical pyramid, the sides of which are sectors of the great circle of the sun, will represent accurately the energy produced by the shrinking of the entire mass. And, in view of the great dimensions of the sun and the formidable array of figures involved in the computation of the energy exerted within the entire sphere, the advantage of considering only the mass covered by a single square foot of the solar surface will be evident. Let *I K S*, Fig. 1, represent the great circle of the sun, *a m a'* the spherical pyramid referred to, and Fig. 2 the said pyramid drawn to a larger scale, its axis being divided into ten equal parts. It is proposed to ascertain what extent of longitudinal contraction of the spherical pyramid *a m a'* is necessary to produce an amount of dynamic energy corresponding with that developed by the radiation from one square foot of the solar surface in a given time. The investigation will be somewhat facilitated if we compute the amount of energy developed by a definite contraction of the sun's radius, say one foot. Let us therefore suppose that *a a'*, the distance of which is $\frac{852,584}{2} \times 5,280 = 2,250,821,760$

Fig. 2

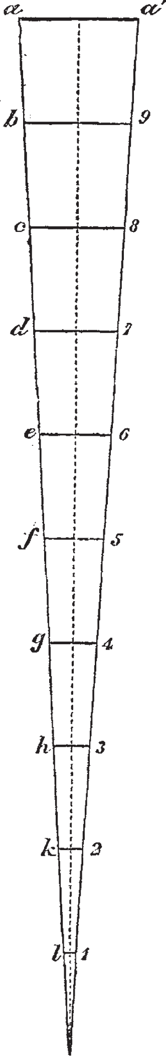
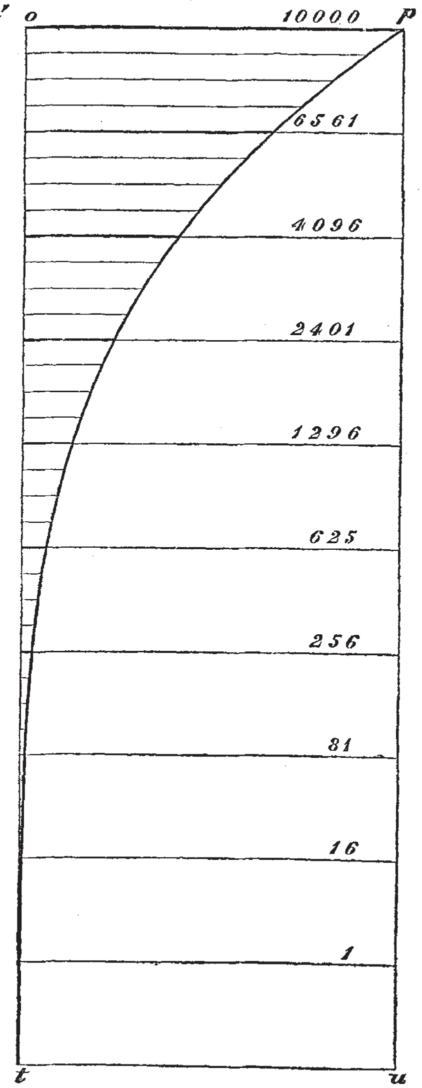


Fig. 3



feet from *m*, has fallen through a space of one foot, the intermediate points *b*, *c*, *d*, &c., participating proportionably in the fall. Assuming that the solar mass remains homogeneous during the contraction, it follows from Newton's demonstration ("Principia," lib. i. prop. lxxiii.) that since a particle just within the circumference of the sphere at *a* is ten times farther from the centre *m* than a particle at *l*, the former will be attracted towards *m* with ten times greater force than the latter. It will be readily perceived that, for a given movement towards the centre, the quantity of matter put in motion at *a* will be greater than at *l*, in the ratio of the squares of *a a'* and *l* 1, or 100 : 1. Hence, in accordance with the demonstration referred to, a given radial depth of the solar mass at *a* will exert a force towards *m*

10 × 100 = 1,000 times greater than an equal radial depth at *l*. But, in computing the dynamic energy developed by the shrinking of the sun, it must be borne in mind that a particle at *a* falls through a distance ten times greater than a particle at *l*. The length of the ordinates of the curve *p t*, Fig. 3, representing the ratio of dynamic energy developed at the respective distances from the sun's centre, has been calculated accordingly. A cursory examination of Fig. 2 can scarcely fail to lead to the conclusion that the mass composing the smaller sections of the spherical pyramid towards the centre of the sphere, will be attracted by the larger mass composing the sections towards the circumference. Newton has disposed of this question by a geometrical demonstration which, considering the form of the attracting mass, and the extreme complication arising from the varying direction and unequal magnitude of the attracting forces, may be regarded as one of the most elegant of his masterly demonstrations of important propositions and theorems. It will be evident on reflection that, unless it can be proved that a particle at *P* is not attracted by any portion of the mass contained within the outer spherical superfices *I K S* and the interior spherical superfices *P p*, the mass composing the sections near the base of the spherical pyramid will exert the disturbing attraction before alluded to. Our demonstration of the energy produced by the attraction of the matter within the sun, during shrinking, falls to the ground, unless it can be shown that every particle composing the spherical pyramid is in perfect repose as regards the attraction exerted by exterior particles. The great geometer thus establishes that repose :—Let *H I K L* be a spherical superfices, and *P* a corpuscle placed within.* Through *P* let there be drawn to this superfices the two lines *H K*, *I L*, intercepting very small arcs *H I*, *K L*; and because the triangles *H P I*, *L P K* are homogeneous, those arcs will be proportional to the distances *H P*, *L P*; and every particle at *H I* and *K L* of the spherical superfices, terminated by right lines passing through *P*, will be in duplicate ratio of those distances. Therefore the forces of these particles exerted upon the body *P* are equal between themselves. For the forces are as the particles directly, and the squares of the distances inversely. And these two ratios compose the ratio of equality. The attractions, therefore, being made equally towards contrary parts, destroy each other. And, by a like reasoning, all the attractions through the whole spherical superfices are destroyed by contrary attractions. Therefore the body *P* will not be anyway impelled by those attractions.

Referring to Fig. 3, let us recollect that the ordinates of the curve *p t* do not indicate the force exerted by mere attraction. As already stated, their length represents the dynamic energy developed at definite distances between the centre and the circumference of the sphere. The energy actually produced is represented by the superficies *o p t*, while the rectangle *o p u t* represents the energy that would be called forth if the force exerted at every point of the axis of the spherical pyramid were the same as that exerted at *a a'*. Our space will not admit of introducing the calculations by which the energy represented by the ordinates of the curve *p t* have been computed. It will be proper, however, to call attention to the fact that the energy exerted at each of the divisions of the base line *o t* is definite; hence the length of the ordinates is exact. Calculations based on the data thus furnished show that the superficies *o p t* is 0·20015 of the superficies *o p u t*.

We have before stated that the want of homogeneity of the solar mass will not materially affect the amount of energy developed by the gravitating force during the sun's shrinking. Referring to the several figures, it will be seen that the energy exerted at a point half way from *m*, viz., ordinate 5, is 0·0625, or $\frac{1}{16}$ of that exerted at *a a'*; and that the energy developed by the mass contained within the spherical pyramid *f m* 5 amounts to only $\frac{1}{31}$ of that developed by the gravitation of the mass contained within the spherical pyramid *a m a'*. Now the volume of the spherical pyramid *f m* 5 represents that of a sphere the diameter of which is one half of the sun, while the spherical pyramid *a m a'* represents the volume of the entire solar mass. The energy resulting from the gravitation of the central spherical mass *P p* being thus only $\frac{1}{31}$ of the energy exerted by the spherical mass *I K S*, it will be perceived that the degree of density of the matter towards the sun's centre will not materially affect the result of our calculations founded on perfect homogeneity.

We may now proceed to ascertain the amount of dynamic energy produced by the assumed shrinking of the axis of the spherical pyramid *a m a'*. Having already demonstrated that the said energy will be 0·20015 of that produced by the gravitation of a homogeneous mass, the section of which is one square foot extending from the surface to the centre, it only remains to determine the weight of one cubic foot at the surface of the sun. The specific gravity of the solar mass being 85·6 pounds per cubic foot, while the sun's attraction is 27·2 times greater than terrestrial attraction, the weight of one cubic foot at the solar surface will be 27·2 × 85·6 = 2328·3 pounds. Multiplying this weight by the sun's radius expressed in feet, we have, 2328·3 = 2,250,821,000 = 5,240,633,000,000, which product, multiplied by 0·20015, shows that the gravitating energy of the matter contained in the spherical pyramid, exerted during a longitudinal contraction of one foot, amounts to 1,048,912,000,000 foot pounds. Dividing this latter product by the solar energy per minute, already stated, we find that 4355 minutes, = 3·024 days will elapse before the energy produced by constant solar radiation equals the gravitating energy exerted during the shrinking of one foot of the solar radius. The length of one year, 365·25 days, being divided by 3·024, we learn that the annual shrinking of the sun's radius amounts to 120·7 feet. The foregoing figures prove that, notwithstanding this apparently great contraction, a period of 1864 years is necessary to diminish the sun's diameter $\frac{1}{10,000}$. It hardly requires explanation that this result is reached by dividing the sun's diameter by 10,000 times the stated annual shrinking.

Helmholtz, in accordance with Laplace's remarkable nebular hypothesis, asserts that the continuation of the original condensation of the matter composing the sun develops an amount of mechanical energy capable of generating sufficient heat to make good the present solar emission. According to his calcutions, the sun's diameter will be reduced $\frac{1}{10000}$ in the course of 2,000 years. The practical data assumed by the eminent physicist being less accurate than those upon which our calculations are based, the discrepancy regarding time, 2,000 years against 1864 years, necessary to effect the stated shrinking of the sun's diameter, may be satisfactorily explained. It will be well to observe that the intensity of the radiant heat will not diminish with the diminished size of the sun. On the contrary, for a given area of the solar surface, the dynamic energy produced by a given rate of shrinking will be increased, since the mass remains the same, while the attraction is inversely proportional to the square of the distance from the centre. But the *rate* will diminish with the contraction of the sphere; hence a shrinking of $\frac{1}{10}$th of the sun's diameter, instead of occupying 1,000 × 1864 = 1,864,000 years, will require somewhat more than 2,000,000 years. At the end of that period the gravitating energy will continue to develop, as at present, an amount of dynamic energy represented by 312,000 thermal units per minute for each superficial foot; but the radiating surface, *i.e.*, the area of the solar disc, will have diminished in the ratio of 10^2 to 9^2.

The present maximum temperature produced by solar radiation on the ecliptic when the earth is in aphelion, being 67·2, while the intensity of radiant heat diminishes as the area of the radiating surface, it follows that, at the end of 2,000,000 years from the present time, the tropical solar intensity will be reduced to $\frac{9^2 \times 67{\cdot}2}{10^2} = 54{\cdot}4°$, unless Prof. Tyndall's opinion is correct, that the earth, in common with the other planets, must "creep in, age by age, towards the sun."* But the pace is no doubt so slow that our calculations will not be seriously affected; hence, applying the foregoing demonstrations to the past, it will be seen that the temperature called forth by solar radiation 2,000,000 years ago must have been, owing to the greater diameter of the sun at that period, about $\frac{11^2 \times 67{\cdot}2}{10^2} = 81°$ within the tropics. Now we are justified in assuming that the increased evaporation of the sea, and the consequent humidity of the atmosphere, modified the stated solar intensity, calling forth the luxuriant flora of past ages, which geology has made us acquainted with. The computed diminution of solar intensity, 67° − 54° = 13°, during the next 2,000,000 years will probably be deemed extravagant by those who do not bear in mind that the computation must be based on

* Sir Isaac Newton, in his demonstrations relating to spherical bodies, supposed these to be composed of an infinite number of spherical superficies the thickness of which he thus defines:—"By the superficies of which I here imagine the solids composed, I do not mean superficies purely mathematical, but orbs so extremely thin that their thickness is as nothing; that is, the evanescent orbs of which the sphere will at last consist, when the number of the orbs is increased, and their thickness diminished without end."

* See "Heat as a Mode of Motion," p. 499.

the assumption that a constant power is being exerted during the stated period capable of developing, as at present, the stupendous energy of 240 millions of foot-pounds in a single minute, for each square foot of the surface of a sphere whose diameter exceeds 850,000 miles. This inconceivable amount of work cannot be performed with a less expenditure than the motive energy developed by the fall of a mass equal to the mass contained in the sun, the weight of which is nearly a thousand times greater than the weight of all the planets of the system. Obviously a *continuous* development of such an amount of energy is physically impossible, since there is a *limit* to the distance through which the weight can fall. Now the foregoing demonstration enables us to determine the said limit, with sufficient exactness to prove that although the efficiency of the great motor, during the past, may be measured by hundreds of millions of years, its future efficiency will be of comparatively brief duration.

Statements relating to the permanency of solar heat, based on the assumption that no diminution has been observed during historic times, have no weight in view of our demonstration showing that a shrinking of $\frac{1}{10}$ of the sun's diameter can only reduce the intensity from 81° to 67°·2, difference = 13°·8, in the course of two millions of years. This period being 500 times longer than "historic times" say 4,000 years, it will be seen that the diminution of the temperature produced by solar radiation, has not exceeded $\frac{13 \cdot 8}{500} = 0 \cdot 027$, or $\frac{1}{37}$ deg. Fah. since the erection of the Pyramids.

It will be proper to observe, before concluding our brief investigation of the source of solar energy, that, the development of heat by the shrinking of the sun, however fully demonstrated, leaves the important question unanswered: how is the heat generated by gravitation within the mass transmitted to the surface? If the matter within the sun is a perfect conductor of heat—a very improbable supposition—that fact alone furnishes a satisfactory answer. Imperfect conductivity, on the other hand, calls for other means of transmitting the energy from within, to make good the enormous loss caused by the external radiation. Besides, the falling of the crust at the rate of ten feet per month, attended by increase of internal pressure, and probably ejection of gaseous matter, together with the disturbance occasioned by contraction at the surface, disclose a mechanism of startling perplexity. But the parting with 312,000 thermal units for each square foot of the solar surface, involving an expenditure of kinetic energy fully 240,000,000 foot-pounds per minute, cannot be made good in that brief space of time, unless the sun shrinks at the rate ascertained by our calculations.

The development of solar energy in accordance with the combustion hypothesis (lately resuscitated by M. E. Vicaire) merits no consideration, while careful investigation has proved the meteoric hypothesis to be untenable. It must be admitted, however, that the mechanical difficulties alluded to, especially those relating to the means of transmitting the heat to the surface of the sun, any temporary local derangement of which must be productive of dark spots for a time, are of such a nature that the absolute certainty of solar radiation may be questioned; nor is evidence wanting to show that the solar mechanism is liable to derangement. History informs us that the great luminary has, during several seasons, partially failed to perform its functions. Herschel states, in his "Outlines of Astronomy," that "in the annals of the year A.D. 536 the sun is said to have suffered a great diminution of light, which continued fourteen months. From October A.D. 626 to the following June a defalcation of light to the extent of one-half is recorded; and in A.D. 1547, during three days, the sun is said to have been so darkened that stars were seen in the day-time." Again, the glacial periods, the ascertained abrupt termination and recurrence of which puzzles the geologist, point to periodical derangement of the solar mechanism in past ages.

J. ERICSSON

EXTRAORDINARY WHIRLWIND IN IRELAND

IN a letter to the *Belfast News-Letter*, Mr. C. J. Webb describes an extraordinary whirlwind which occurred in the district around Randalstown, about six miles N.W. of Antrim, near the shores of Lough Neagh, on the 25th of August last. The same phenomenon was witnessed about an hour and a half earlier the same evening at Banbridge, about seven miles S.W. of Dromore. It was first seen near Randalstown about 5 P.M., between that place and Toome, moving rapidly up Lough Neagh from the south, and presenting the appearance of a defined column of spray and clouds, whirling round and round, and not many yards in breadth, while at its base the water was lashed into a circle of white foam. It was next heard of in the neighbourhood of Staffordstown, about a mile from the lake, where it partially unroofed two houses, and damaged any trees or crops which happened to be in its course. From this point it travelled in a straight line for Randalstown, about three miles distant. It passed across a field close to Mr. Webb's house, levelling eight haystacks, and carried a considerable part of the hay up into the air out of sight. The breadth of the storm could be accurately ascertained at this point, and must have extended about thirty yards, as stacks remained unruffled at either side, while those between were thrown down and carried away or scattered about. Everything it lapped up was whirled round and round, and carried upwards in the centre, while dense clouds seemed to be sucked down on the outside, and came close to the earth. Both before and after there was lightning and incessant peals of thunder; but there was no rain till some time afterwards. Mr. Webb next observed its track in a hollow, some three hundred yards further on, where it knocked down a haystack, and then plunged into a wood of fine old Irish oaks. Here it tore numerous branches and limbs from the trees, carrying some along with it, and throwing others to the ground. One noble tree in the centre of the wood seems to have been a peculiar mark for its vengeance, although it would have been completely protected from any ordinary storm, owing to its position. It next passed across a corner of Shane's Castle demesne. Some who were at a short distance from this point describe its approach as causing considerable alarm. It was accompanied by a wild rushing noise, and the crashing of the trees and branches could be heard becoming louder and louder as it advanced. It crossed the valley over the railway viaduct, close to Randalstown, fortunately avoiding the village. It here presented the appearance of a vast whirling column of leaves and branches, mingled with clouds which looked like smoke.

The railway station next suffered, innumerable slates and two and a half cwt. of lead being torn from the roof in an instant. A great part of the railings surrounding the gardens was torn up, and an iron bar one inch thick, belonging to the gate, was bent to an angle of sixty degrees. A small shed at the rear of the station was unroofed, rafters and slates being hurled to the ground. What will give some idea of the excessive pressure of the wind, is the fact that three boards of the flooring of the waiting-room were forced up, owing to the wind finding an entrance to a cellar underneath, though the only aperture was a round hole about one foot in diameter. All this was the work of a few moments. The storm then passed away, leaving comparative calm behind. It next crossed an adjacent bog, scattering the turf in all directions. The last place Mr. Webb heard of its having visited was a farm house about three miles from Randalstown, between Antrim and Ballymena. It would be interesting to ascertain whether it travelled across to the sea-coast.

NOTES

THE British Association Committee on Mathematical Tables, of which Prof. Cayley is the chairman, has determined to tabulate the Elliptic Functions, or more accurately, the Jacobian Theta Functions, which are the numerators and denominators of the former, and their logarithms. The tables, which are of double entry, will therefore give eight tabular results for each