Does gravity correct gauge couplings?

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Whether various models of quantum gravity predict observable effects is a matter of dispute. Toms has argued[1] that there are quantum gravity corrections to the energy dependence of the electric charge in quantum electrodynamics (QED) that depend quadratically on the energy, and cause the electric charge to vanish at high energies. This conclusion was based on a background field calculation using a cut-off thought to be related to energy. We argue that scattering processes cannot have such a quadratic energy dependence. Hence the quadratic correction[1] cannot apply to a charge that is physically measurable, and does not lead to asymptotic freedom in QED.

In the absence of gravitation, couplings in gauge field theories (such as the electric charge in QED) vary logarithmically with energy due to quantum corrections, with that in QED increasing. It has been suggested[2] that quantum gravity would make a correction that depends quadratically on energy, causing all gauge couplings to vanish at high energies (even that in QED), the property known as asymptotic freedom. An observation of this effect would have profound implications for the unification of gravity with the gauge interactions, as it would cause the latters' couplings to drop very rapidly at scales above the expected grand unification scale, 10^{16} GeV.

However, several confusing and contradictory calculations of such gravitational corrections have appeared subsequently, ranging from explicit gauge dependence[3] to the absence of such terms in dimensional regularization[4], contradicting results in other approaches[5].

A covariant approach to the problem was taken recently by Toms[1], based on a gauge-invariant heat-kernel regularization using a generalized background-field method. It was claimed that the quadratic energy dependence of the gravitational contributions to the electric charge was confirmed, and an additional logarithmic energy dependence was found, proportional to a positive cosmological constant. This calculation used a proper time cut-off τ_c related to an energy cut-off E_c, which was then identified with the energy E at which the renormalized electric charge $e(E)$ is evaluated. The resulting corrections would render asymptotically free the corresponding QED running coupling, and similar results would hold for non-Abelian gauge theories.

The disagreements[1–5] raise the question of whether the claimed gravitational corrections to gauge couplings are physical, and specifically the question of whether the appearance of a quadratic cut-off dependence actually signals the appearance of an asymptotically-free coupling in physical processes: the previous calculation[1] could be absorbed into a trivial charge renormalization if E_c and E were not identified.

Physical measurements—for example, of the electric charge—are derived from on-shell scattering amplitudes (S-matrix elements). These are invariant under local redefinitions of fields, as shown in the equivalence theorem[6,7]. As we now show, the equivalence theorem implies that energy-dependent modifications of gauge couplings, such as those discussed above, cannot affect S-matrix elements, and hence are not relevant for asymptotic freedom or the unification of gauge interactions with gravity—for example, in string theory.

The equivalence theorem[6,7] asserts that if one redefines a generic field φ by $\varphi \to \Phi = \varphi + F(\varphi)$, where $F(\varphi)$ is a local, gauge invariant combination of φ and its derivatives that does not influence the mass-shell condition for the particle associated with φ, and the correlation functions of F with itself and/or φ do not have poles corresponding to massless particles, then the S-matrix in the transformed theory is the same as in the original theory.

As the claimed gravitational corrections to gauge couplings may be removed by a field redefinition satisfying the conditions of the equivalence theorem, these corrections can have no physical effects on on-shell scattering processes. This also explains the discrepancies described above, including the apparent dependences on the gauge-fixing parameter and the regularization scheme, because the quantities being calculated were not physically measurable.

From the point of view of an effective action, the E^2-dependent terms[1–5] correspond to higher-derivative terms of the form: $\frac{b}{M_{\mathrm{P}}^2}\int d^4x F_{\mu\nu} \Box F^{\mu\nu}$, where b is a dimensionless numerical constant, $M_{\mathrm{P}} \approx 10^{19}$ GeV is the Planck energy scale, $F_{\mu\nu}$ is the electromagnetic field strength and $\Box$ is a covariant second derivative. Terms of the form $(\nabla_\rho F_{\nu\mu})^2$ can easily be cast in the same form, which is the only independent higher-derivative combination that is quadratic in the field strength and in space-time derivatives, and terms of the form $\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu}$ vanish thanks to the cyclic permutation identity. It is straightforward to see that the coefficient b above can be changed by the following local field redefinition of the gauge potential, A_μ, which respects the criteria of the equivalence theorem outlined above: $A_\mu \to \tilde{A}_\mu = A_\mu + \frac{c}{M_{\mathrm{P}}^2}\nabla^\nu F_{\nu\mu}$, where $\tilde{A}_\mu$ denotes the redefinition of A_μ, c is an arbitrary numerical constant and ∇_μ denotes a gravitationally-covariant derivative. All corrections to the photon propagator can be removed by such local redefinitions, and no terms with any number of derivatives that are bilinear in the gauge potential A_μ, such as $(\nabla_\rho \ldots \nabla_\lambda F_{\alpha\beta})^2$ and so on, have any effect on on-shell scattering amplitudes, as is well known in string theory[8].

It follows that there are no relevant power-law gravitational corrections to the physical electric charge, no asymptotic freedom in QED, and no effect on the comparison between gauge and gravitational interaction strengths in string unification scenarios, as had been shown explicitly in the context of open strings[9,10], without appeal to the equivalence theorem. This observation can be extended to terms of higher orders in the effective gauge theory, but does not extend to the term proportional to the cosmological constant that depends logarithmically on the energy scale, which cannot be absorbed by a local field redefinition.

A related point of view was made recently in the context of a scalar field theory with a $\lambda\Phi^4$ interaction, where it was argued that power-law corrections due to quantum gravitational interactions do not signify a running of the coupling constant λ that can be measured physically[11]. In the context of our discussion, this is another illustration of the equivalence theorem.

We conclude by restating our principal conclusion: the equivalence theorem implies that S-matrix elements are unaffected by higher-order derivative corrections that are quadratic in the gauge fields, and hence that the measurable electric charge does not exhibit a quadratic energy dependence leading to asymptotic freedom, as suggested by Toms[1].

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Toms replies

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In a previous paper[1], I used the gauge-invariant background-field method to calculate a contribution to the renormalization of electric charge due to quantum gravity that was quadratically dependent on an energy cut-off. This was done by taking a constant electromagnetic background field, F. The result was used to support the original suggestion of Robinson and Wilczek[2] that quadratic divergences could lead to asymptotic freedom. This result has been criticized by Ellis and Mavromatos[3] and their present submission[4] gives a shortened and altered discussion of their viewpoint.

The implication of Ellis and Mavromatos[4] is that the quadratic divergence that I found[1] is related to the dimension-six operator $F\partial^2F$. This is not the case. The quadratic divergence proportional to F^2 that was calculated has nothing to do with $F\partial^2F$. I agree with these authors that the coefficient of $F\partial^2F$ can be altered by a field redefinition and cannot affect any physically measurable quantity; by taking F to be constant as I did, the dimension-six operator cannot occur. What Ellis and Mavromatos[4] argue is that the dimension-six operator cannot affect the renormalization (and hence running) of charge, and nothing more; their criticism has no direct bearing on the calculations reported by Robinson and Wilczek[2] or myself[1].

The reason why I do not now believe that the quadratic divergences contribute to a running electric charge has to do with not properly identifying a physically measurable definition of the charge. (I am grateful to J. Donoghue (personal communication) for pointing this out.) This should indeed follow from an S-matrix calculation but no one, including Ellis and Mavromatos, has attempted this calculation for Einstein–Maxwell (or Yang–Mills) theory. (A related S-matrix calculation has been done[5] for a non-gauge field.) Instead the background-field method in one form or another has been used. A physical definition of charge can be considered completely within the background-field method where it can be shown[6] that the quadratic divergences do not contribute to the running electric charge, and only logarithmic divergences do so contribute. Although it does not appear that the quadratic divergences lead to a running of gauge coupling constants, it is still possible[7] that quantum gravity can lead to asymptotic freedom if there is a positive cosmological constant; however, the running is only logarithmic, not quadratic, and is not as interesting phenomenologically.

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